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A Two Step Model for An Urban Transit Problem

by

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ABSTRACT

The purpose of this paper is the introduction of new theoretical solutions about urban transportation. During the last two decades there have been many contributions, such as those of Patrikson [3] , Thomas [2] , Bennet [1] in order to mention a few of them. They have developed models using only heuristic approaches and then with important limitations.

In this paper, we propose a linear model which is based on the assumption that the characterization solutions depend on the existence or nonexistence of cycles. From this idea, we develop an algorithm which finds all the solutions of the problem.

The mathematical formulation corresponding to it depends on the set of linear equations and on the matricial formulation of the problem. The obtained numerical results show that the proposed model might manage a great number of data. Moreover it may be of interest, in concrete for the transit behavior of real cities.

1. INTRODUCTION

Planning of urban transit and transport, as well as the analysis of related problems (for example Improta [4] , Allshop[5], [6], [7], etc.) motivated an important research branch in Applied Mathematics.

The basic theory of the assignment in transport has been developed extensively by Patrikson[3], Bennet [1], and Thomas [2]. The main assumptions considered in our network are :

1-1 The movement of each vehicle is performed between an initial and a final node. This situation generalizes the situation of the other classical consideration of having the source and the sink with the important result of Ford-Felkerson [8] (see also Rockefeller [9]).

In the path of movement of each vehicle, there exist passing compulsory nodes, which the drivers must choose. They restrict the flow of the entire network.

1-2 The number of vehicles that might pass through the compulsory node is not restricted.

1-3 The vehicles found in the initial nodes_pass through intermediate nodes and then all arrive at the final destination.

1-4 All drivers have knowledge of all the characteristics of the traffic net and transit graph.

1-5 The trip times over all the paths used are less than those possible of paths which might be experienced by a vehicle in any other unspecified way.

1-6 The model is static in time and deterministic.

The figure I shows the expression of the graph of transit in the case n=3

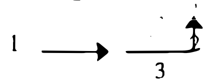


fig. I

2. INTERPRETATION AND PROBLEM MODEL

Traditionally, in the general theory of transportation, its flow in between part (initial nodes of the transit net or graph) and as final destination (final nodes of such graph). A different approach is that of Ford-Felkerson [8]. Now if we introduce among all the ports and destinations an intermediate step, which is represented by deposits (compulsory passing nodes), then we obtain a new transportation problem, known as the problem of two step transportation. The treatment of this situation is rather important, since it will allow the solving of new trivial problems of urban planning. This model was introduced by Marchi and Tarazaga in [15].

Formally, a problem with m ports, n deposits and p, destinations, in which it is assumed that

- The totality of the amount available in the ports is distributed.
- In the deposits the capacity is not bounded
- There is no accumulation in the deposits

might be modeled as follows :

r_i : capacity of the port i.

t_k :capacity of the destination k.

x_{ij}^1 :units to be transported fron port i to the deposit j, which are assumed to be a real non negative number.

x_{jk}^2 :units to be transported from deposit j to the destination k and with a conservation law $\sum_{i=1}^{m} r_i = \sum_{k=1}^{p} t_k$

and

(1) $\sum_{j=1}^{n} x_{ij}^1 = r_1$ $\quad i = 1,\ldots,m$

(2) $\sum_{j=1}^{n} x_{jk}^2 = t_k$ $\quad k = 1,\ldots,p$

(3) $\sum_{i=1}^{m} x_{ij}^1 - \sum_{k=1}^{p} x_{jk}^2 = 0$ $\quad j = 1,\ldots,n$

The description of the problem is completed with the formulation of the objective function or cost function

$$f(x) = c^1x^1 + c^2x^2 = \sum_{i,j} c_{ij}^1 x_{ij}^1 + \sum_{j,k} c_{jk}^2 x_{jk}^2$$

which will be minimized or maximized depending on the economic content described by the transportation model.

3. MODEL ANALYSIS

The problem of two step transportation, described by 1), 2) and 3) together with the payoff function is a linear programming problem.
If the cost function is non linear it will become a non linear program ; see Mangarasian [13] and Farkas [14].
For the analysis in the general context related to the existence of optimal solutions it is possible to use the traditional result of Farkas[14], or Tucker, or Gale, etc.(See for example Mangarasian [13])
But given the general characteristics of the model, we will study the model from a more adequate perspective, allowing us to find a general algorithm for finding the extremals and therefore to obtain in an easier way the set of optimal solutions.
In the first place, note that it is trivial to prove:

Proposition 3-1. The set of possible solutions of 1), 2), 3) is a convex polyhedron.

3.2. MATRICIAL REPRESENTATION

The matricial representation of the problem, has the form:

$$Ax = b$$

where A is the matrix of order $(m+p+n)\times[(m+p)n]$ given by:

$$a_{ij} = \begin{cases} 1 & j = (i-1)n+1,\dots,n \\ 0 & otherwise \end{cases}$$

for $i = 1,\dots,m+p$ and

$$a_{ij} = \begin{cases} 1 & j = i-(m+p)+kn & k = 0,\dots,n-1 \\ -1 & j = i-(m+p)+(m+k)n & k = 0,\dots,p-1 \\ 0 & otherwise \end{cases}$$

for $i = m+p+1,\dots,m+p+n.$

Therefore

$$A = \begin{array}{c} \overbrace{\qquad\qquad\qquad\qquad}^{mn}\ \overbrace{\qquad\qquad\qquad\qquad}^{pn} \\ \left[\begin{array}{cccc|cccc} 1 & 1\dots1 & \cdot & 1\dots1 & & & & \\ \hline & & & & 1\dots1 & \cdot & \cdot & 1\dots1 \\ \hline 1\ \ddots\ 1 & 1\ \ddots\ 1 & \cdot & 1\ \ddots\ 1 & -1\ \ddots\ -1 & -1\ \ddots\ -1 & \cdot & -1\ \ddots\ -1 \end{array}\right] \begin{array}{l} \}\,m \\ \}\,p \\ \}\,n \end{array} \end{array}$$

$$b = \begin{bmatrix} r_1 \\ \cdot \\ r_m \\ t_1 \\ \cdot \\ t_p \\ 0 \\ \cdot \\ 0 \end{bmatrix}$$

From the descriptions of the matrix A, it is possible to compute the dimension of the subspace generated by its row vectors in the matrix, from which we obtain

Proposition 3-2-1 The rank of A is $m+p+n-1$.

Proof. Let

$$F_1 = \left(a_{i1}, \ldots, a_{i,(m+p)n}\right), F_1 \in \mathbb{R}^{(m+p)n}$$

$$S_F = \left\{ v / v = \sum_{i=1}^{m+p+n} K_i F_i \quad , K_i \in \mathbb{R} \right\}$$

Since $r(A) = \dim S_F$, we will prove that dim $S_F = m + p + n - 1$. In order to show it we will prove that $B = \left\{F_2, \ldots, F_{m+p+n}\right\}$ in a base of S_F.

Indeed

3-2-2-1 B is a set of generator of S_F since

$$F_1 = \sum_{i=m+1}^{m+p+n} F_i - \sum_{i=2}^{m} F_i$$

and F_i (for $2 \le i \le m+p+n$) it is possible to trivially write it as a linear combination of elements of B.

3-2-1-1 B is a set of linearly independent vectors because if one takes

$$\sum_{i=2}^{m+p+n} h_i F_i = 0$$

it turns out a system of $m+p+n-1$ unknowns and $(m+p)n$ equation of the form :

$$\sum_{i=2}^{m+p+n} h_i a_{i1} + \sum_{i=m+p+1}^{m+p+n} h_i a_{i1} = 0 \qquad \text{for } i = 1, \ldots, (m+p)n$$

Splitting the system into blocks of n equations from the first block of n it turns out that

$$h_{m+p+1} = \ldots = h_{m+p+n} = 0$$

and from the latter

$$h_2 = 0$$

and consequently

$$h_{m+p} = 0$$

As a consequence of **3-2-1-1** and **3-2-1-2** the proposition is proved.
As a final part of this paragraph we wish to point out, for the use given in the next considerations, A can be partitioned as

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \\ U_1 & U_2 \end{bmatrix}$$

3-3 EXTREMAL CHARACTERIZATION

As it was said, the model is linear, and as a consequence, the characterization of the extremals of the convex polyhedron of possible solutions (op. cit. prop. 3-1) will hold, if it is not empty, to the obtention of

the hull optimal solution (see Dantzig G.B. [11]). Given the particular characteristic of the problem, which is reflected in the matrix A, we conjecture that the extremal characterization can be derived from itself.
On this line, as a first result we present :

Proposition 3-3-1 Each extremal of the convex polyhedron of admissible solutions of the problem given by 1), 2) and 3) has at most $m+p+n-1$ positive elements.

Proof :
Let $e \in S$ (op. cit. prop. **3-2-1**), e extremal. Then e satisfies

$$Ae = B$$

From the proposition **3-1**, let B be a non singular submatrix of A. of order $m+p+n-1$.Consequently

$$[B,N]\begin{bmatrix} e_B \\ e_N \end{bmatrix} = b$$

and, from the general theory of matrix algebra, if we take $e_N = 0$, at most the $m+p+n-1$ components of e_B are positive.

Proposition 3-3-2. If $s = \left(s^1_{11},\ldots,s^1_{mn},s^2_{11},\ldots s^2_{np}\right)$ is an element of S, such that for each $1 \leq i \leq m$, $1 \leq k \leq p$, there exists a unique j $(1 \leq j \leq n)$ such that s^1_{ij} and s^2_{jk}, then s is extremal.

Proof :
Let s be a solution such that it satisfies the hypothesis and assume that s is not extremal.Then

3-3-2-1 there exist s' and s'' solutions, such that s is a convex combination

$$s = \lambda\ s' + (1-\lambda)s'' \qquad \text{with } 0 \leq \lambda \leq 1$$

3-3-2-2 $\quad (\forall i) \quad (\exists j) \quad \left[\left(s^1_{ij} = r_1\right) \wedge \left(s^1_{i1} = 0 \ \text{ if } \ 1 \neq j\ \right)\right]$

3-3-2-3 $\quad (\forall k) \quad (\exists j) \quad \left[\left(s^2_{jk} = t_k\right) \wedge \left(s^2_{hk} = 0 \ \text{ if } \ h \neq j\ \right)\right]$

From **3-3-2-1, 3-3-2-2** and **3-3-2-3** it results that

$$s = s' = s''$$

contradicting the assumption that s is not extremal.
In order to characterize all the extremals, we oberve that if we have a solution :

$$\left(s^1_{11},\ldots,s^1_{mn},s^2_{11},\ldots,s^2_{pn}\right)$$

of the two step problem, indeed we have the solution of each one of the two linear problems defined by A_1 and A_2, respectively.
In figure II the reader might visualize a graphic arrangement for such a situation, where in the inferior part we have exchanged rows by columns.

$$\begin{array}{cc} & j \\ i & \boxed{s^1_{ij} \quad\quad\quad\quad} \end{array}$$

$$\begin{array}{cc} k & \boxed{s^2_{jk} \quad\quad\quad\quad} \end{array}$$

fig. II

From such an arrangement of Fig. II, which we will use each time that we refer to a solution, it was possible to find a characterization of the extremals. In order to illustrate such point we will consider an example.
For the problem

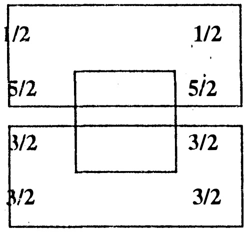
$$\left[\begin{array}{cccccc|cccccc}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\hline
1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1
\end{array}\right] \cdot \begin{bmatrix} x^1_{11} \\ x^1_{12} \\ x^1_{13} \\ x^1_{21} \\ x^1_{22} \\ x^1_{23} \\ x^2_{11} \\ x^2_{21} \\ x^2_{31} \\ x^2_{12} \\ x^2_{22} \\ x^2_{32} \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

are solutions

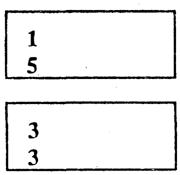
$$\boxed{\begin{array}{ll} 1 & \\ 2 & 3 \end{array}}$$

$$\boxed{\begin{array}{ll} 3 & \\ & 3 \end{array}}$$

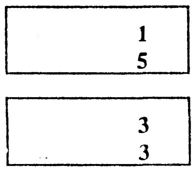
and



from which the first is extremal, and the second is a convex combination of



and



We note if we call support of the solution to the set

$$S = \left\{ (i,j,k) / x^1_{ij} \rangle 0 \wedge x^2_{jk} \rangle 0 \right\}$$

we see that such solutions are possible and they behave in different ways on the possible supports. In the second of them there exists a support subset on which it is possible to have a path beginning from one positive entry (from now on vertex), arriving to such an entry

Definition 2-3-3 Let $S = (s^1, s^2)$ be a solution of the problem, a cycle in the support of s is a sequence of indexes

$$i_{q_1} j_{q_1} \dots j_{q_i} 1_{q_i} i_{q_n} j_{q_n} \dots i_{q_1}$$

in the support of s, where it is possible that :

$$I) j_{q_i} \neq j_{q_n} \quad \wedge \quad 1_{q_i} = i_{q_n} = k_{q_n}$$

$$II) j_{q_i} = j_{q_n} \quad \wedge \quad 1_{q_i} = k_{q_n} \wedge 1_{q_n} = i_{q_n}$$

We remind that

Theorem 3-3-4. If a solution does not have cycles in the support there necessarily exists

(i,j,k) **such that** $\quad x^1_{ij} \rangle 0 \quad \wedge \quad x^1_{ij'} = 0 \quad$ **for** $\quad j \neq j'$

or

$\quad x^2_{jk} \rangle 0 \quad \wedge \quad x^2_{j'k} = 0 \quad$ **for** $\quad j \neq j'$

The proof is trivial using the contrareciprocal proposition. The fact that the extremals do not contain cycles has a reason :

Theorem 3-3-5. A solution of the problem defined by 1), 2), 3) is extremal if and only if it does not contain cycles in its support.

Proof .
The condition is necessary. Indeed, let e be an extremal of the problem and assume that e contains a cycle in its support. Let

$$V = \{(h_1, l_1), \dots, (h_u, l_u)\}$$

be the set vertices of such a cycle and besides suppose that the cardinality of such is even (if this were not true, it is always possible to consider a subset of such a set of even cardinality which determines the same cycle). Let s' and s'' be the solutions which are obtained using e in the following way :

$$s_{h_q l_q} = \begin{cases} e_{h_q l_q} & if \quad (h_q, l_q) \notin V \\ e_{h_q l_q} + \varepsilon & if \quad q \ \ is \ \ odd \\ e_{h_q l_q} - \varepsilon & otherwise \end{cases}$$

$$s_{h_q l_q} = \begin{cases} e_{h_q l_q} & if \quad (h_q, l_q) \notin V \\ e_{h_q l_q} + \varepsilon & if \quad q \ \ is \ \ non \\ e_{h_q l_q} - \varepsilon & otherwise \end{cases}$$

for $\qquad \varepsilon \le \{min \ \ e_{h_q l_q} \}$

Then $\qquad e = \frac{1}{2} s' + \frac{1}{2} s''$

contradicting the assumption that e is extremal.
In order to prove that a solution of the problem defined by **1), 2), 3)** which has no cycles in its support is extremal we will proceed by induction on the number $m + p$.
Consider in the first place $m + p - 2$. Let $s = (s^1_{11}, \dots, s^1_{1n}, s^2_{11}, \dots, s^2_{n1})$ be a solution with the hypothesis conditions and assume that s is not an extremal. Therefore

T.3-3-5-1 $\qquad s^1_{1j} = s^2_{j1} \qquad j = 1, \dots, n$

T.3-3-5-2 There exists a unique j such that

$$s^1_{1j} = r_1 \qquad \textbf{and} \qquad s^2_{j1} = t_1$$

T.3-3-5-3 $\qquad s = \lambda s' + (1 - \lambda) s'' \qquad$ (s' **and** s'' **solutions**).

From **T. 3-3-5-2** and **T. 3-3-5-3** it turns out :

$$s = s' = s''$$

contradicting the assumption that s is not extremal.
Assume now that the assumption is valid for $m+p$. For the case $m+p+1$ (which in terms of the model means that we add a port or a destination), let $s=\left\{s_{11}^{1},\ldots,s_{mn}^{1},s_{11}^{2},\ldots,s_{n,p+1}^{2}\right\}$ be a solution with the hypothesis conditions and assume that s is not an extremal :

T.3-3-5-4 $s=\lambda\ s'+(1-\lambda)s''$ (s' **and** s'' **solutions**).
Besides, without loss of generality, suppose
T.3-3-5-5 $s_{ij}^{1}=r_i$.
From **T. 3-3-5-4** and **T.3-3-5-5** it holds true
T.3-3-5-6 $s^{1}=s'^{1}=s''^{1}=0$ **for** $j\neq j'$
Since $s_{1j}^{1}=0$ (respectively $s''^{1}_{ij}=0$) for $j\neq j''$, and s' (respectively s'') is a solution. Necessarily we have :
$S'^{1}_{ij}=r_i$ (respectively $s''^{1}_{ij}=r_i$) .
When we eliminate the row i, we obtain a problem of less dimension. In it by induction hypothesis, any solution without cycles is extremal. Adding the eliminated row, we get a solution s with no cycles which is identical to s' and s'', contradicting the assumption.

3-4. DETERMINATION OF SOLUTIONS

We have just solved the characterization, of the extremals of the transportation problem in two steps, using the cycles. Now remains the computation problem. With such a purpose we propose an algorithm which generalizes the powerful Jurkar and Ryser algorithm of the classical transportation.
This is determined as follows :

Algorithm to determine the extremal in the transportation problem of two steps.

Step 0 : to determine the variables
Step 1 : 1.1 To select a 3-uple (i,j,k)

1.2 Determine if it forms a cycle with the already chosen.

1.2.1 If it is not, compute $\min(r_i,t_k)$.

1.2.1.2 Assign such a min to x_{ij}^{1} and x_{jk}^{2}.

1.2.2 If it is yes, go back to 1.1.

Step 2 : If r and t are zero for all the values of i,k. Stop the algorithm.
In such a context, a rather important result is

Theorem 3-4-1. The previous algorithm converges to solution of the two-step transportation problem.
The proof is not given since it is trivial.
Corollary 3-4-2 The product solutions by the algorithm are extremals to the two-step transportation problem.
Proof. Trivial using theorem **3-2-4-1** and **3-2-3-3.**

Theorem 3-4-3 Given an extremal of the problem defined by 1), 2), 3), it is always possible to construct it by using the algorithm.

Proof.
We will prove the preceding by induction on $m+p$
3-4-3-1 For $m+p=2$. Since the problem is feasible $r_i=t_i$, it turns out that the proof in the case is trivial.
3-4-3-2 Assume that the property is valid for problems with dimension less or equal to $m+p$ $(m+p$ fixed $)$.
Let e be an extremal of the problem. Because e does not have any cycles in its support, it is possible to assume, without loss of generality, that there exist i_0, j_0 such that $x_{i_0j_0}=r_{i_0}$ and $x_{i_0j}=0$ for $j\neq j_0$.
Then it might happen :
3-4-3-2-1 $\quad r_{i_0}\leq t_k$ for any k
making :

$${}^1x^1_{i_0j_0}=r_{i_0}=0$$

$${}^1x^2_{j_0k_0}=x^2_{j_0k_0}-x^1_{i_0j_0} \qquad \text{for } k_0 \text{ such that } x^2_{j_0k_0}\rangle 0$$

$${}^1t_{k_0}=t_{k_0}-x^1_{i_0j_0}$$

$${}^1t_k=t_k \qquad k\neq k_i \quad ; \quad {}^1x^2_{jk}=x^2_{jk} \qquad \text{for } j\neq j_0, k\neq k_0.$$

$${}^1r_i=r_i \qquad i\neq i_i \quad ; \quad {}^1x^1_{ij}=x^1_{ij} \qquad \text{for } j\neq j_0, i\neq i_0.$$

We obtain a problem of less or equal dimension of $m+p$, for which

$${}^1e=\left\{{}^1x^1_{ij},{}^1x^2_{jk}\right\} \qquad \text{for } i\neq i_0$$

is an extremal, where by induction hypothesis is constructed by the reconstruction of the algorithm.
Now taking i_0 and k_0 we have that $r_{i_0}\langle j_{k_0}$, then for i_0j_0 making :

$$m=r_{i_0}=x_{i_0j}$$

we obtain

$$x^1_{i_0j_0}={}^1x^1_{i_0j_0}+m$$

$$x^2_{j_0k_0}={}^1x^2_{j_0k_0}+m$$

and in this way we reconstruct e using the algorithm.

3-4-3-2-2. there exists k_0 such that $x^2_{j_0k_0} \langle x^1_{i_0j_0}$.

Let

$^1x^2_{j_0k_0} = x^2_{j_0k_0} - x^2_{j_0k_0} = 0$

$^1x^1_{i_0j_0} = x^1_{i_0j_0} - x^2_{j_0k_0}$

$^1r_{i_0} = {}^{\prime}r_{i_0} - x^2_{j_0k_0}$

$^1t_{k_0} = t_{k_0} - x^2_{j_0k_0}$

$^1t_k = t_k \qquad k \neq k_0$

$^1r_i = r_i \qquad i \neq i_0$

$^1x^1_{ij} = x^1_{ij} \qquad j \neq j_0, i \neq i_0$

$^1x^2_{jk} = x^2_{jk} \qquad j \neq j_0,\ k \neq k_0$

Then it might happen :

3-4-3-2-2-1 $\quad {}^1t_{k_0} = 0$ (fig III). In such case we get a problem of dimension less or equal to $m+p$ for which :

$$e^1 = \left\{ {}^1x^1_{ij}, {}^1x^2_{jk} \right\} \qquad i,j,k \neq k_0$$

is an extremal, which by induction hypothesis, is computed by the reconstruction of the form of the algorithm.

Making

$$m = min\left\{ r_{i_0}, t_{j_0} \right\}$$

$$x^1_{i_0j_0} = {}^1x^1_{i_0j_0} + m$$

$$x^2_{j_0k_0} = {}^1x^2_{j_0k_0} + m$$

Now taking $i_0j_0k_0$ we conclude using e' to reconstruct that e was obtained using the algorithm.

3-4-3-2-2-2. $\ {}^1t_{k_0} \rangle 0$ (fig IV)

Suppose ${}^1x^1_{i_0j_0} \langle {}^1x^2_{j_0k}$ for each $k \neq k_0$ such that ${}^1x^2_{j_0k_0} \rangle 0$.

Making

$^2x^1_{i_0j_0} = {}^1x^1_{i_0j_0} - {}^1x^1_{i_0j_0}$

$^2x^2_{j_0k_1} = {}^1x^2_{j_0k_1} - {}^1x^1_{i_0j_0}$

$^2t_{k_1} = {}^1t_{k_1} - {}^1x^1_{i_0j_0}$

$^2r_i = {}^1r_i \qquad i \neq i_0$

$^2t_k = {}^1t_k \qquad k \neq k_1$

$$^2x^1_{ij} = {}^1x^1_{ij} \qquad j \neq j_0,\ i \neq i_0$$
$$^2x^2_{jk} = {}^1x^2_{jk} \qquad j \neq j_0,\ k \neq k_1$$

we obtain a problem of dimension at most $m+p$ for which

$$^2e = \left\{ ^2x^1_{ij}, {}^2x^2_{jk} \right\} \qquad i \neq i_0,\ k \neq k_0$$

is an extremal which by induction hypothesis is a consequence of the algorithm.

Making

$$m_1 = min\left\{ ^1x^1_{i_0j_0} = r^1_{i_0}; {}^1x^2_{j_0k_0} = t_{k_0} \right\}$$
$$\uparrow x^1_{i_0j_0} = {}^2x^1_{i_0j_0} + m_1$$
$$\uparrow x^2_{j_0k_1} = {}^2x^2_{j_0k_1} + m_1$$

results

$$\uparrow x^2_{j_0k_1} = x^2_{j_0k_1} \quad \text{and} \quad \uparrow x^1_{i_0j_0} < x^1_{i_0j_0}$$

Repeating the previous procedure with

$$m_2 = min\left\{ x^1_{i_0j_0}; x^2_{j_0k_0} \right\}$$
$$\uparrow x^2_{j_0k_0} = {}^1x^2_{j_0k_0} + m_2 \quad ; \uparrow x_{i_0j_0} = {}^1x^1_{i_0j_0} + m_2$$

we reconstruct e.

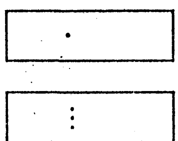


fig. III

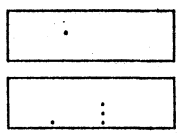


fig. IV

4. CONCLUSIONS

We have developed a mathematical model for the described problem. We have proposed a theoretical solution to it and we have developed an algorithm for its solution. Up to now we have obtained some preliminary numerical results, which show their effectivity and applicability.

Such an algorithm was written in C language. We have spent some time on the numerical efficiency analysis.

We will in the near future compare and extend our results related to other similar transportation models, as for example Marchi [15].

5) BIBLIOGRAPHY

[1] Bennett, L. "The existence of eqivalent mathematical programs for certain mixed equilibrium traffic assignment". European Journal of Operational Research, vol. 71, Dec. 93.

[2] Thomas, R. "Traffic assignment techniques" Avebury Technical. England 1991.

[3] Patrickson, M. "A unified description of iterative algorithms for traffic equilibria" European Jouranl of Operational Research, vol. 71, Dec. 93.

[4] Bruno, G., Improta, G. "Individual controlled functions : Optimal design". European Journal of Operational Research, vol. 71, N 2, pp. 222-234, 1993.

[5] Allshop, R. "Network models in traffic management and control". Transport Review, vol. 3, N 2, pp. 157-182, 1983.

[6] Allshop, R. "Estimating the traffic capacity of a signalized road junction". Trasnpn. Res., vol. 6, 245-255, 1972.

[7] Allshop, R. "Delay at a fixed time signal-I : Theoretical analysis". Transportation Science, vol. 6, N 3, pp. 260-285, 1972.

[8] Ford, L.R., Fulkerson, D.R. "Maximal flow through a network" Canad. J. Math.8, pp. 339-404, 1956.

[9] Rockeffeller, R.T. "Network, Flows and Monotropic Optimization". John Wiley & Son 1984.

[10] Marchi, E., Tarazaga, P. "Two step transportation problem" Relatorio Interno. Universidad de Campinhas, Brazil 1977.

[11] Dantzig, G.B. "Linear programming and extensions" Princeton University Press. Princeton N.J. 1962.

[12] Hu, T.C. "Integer programming and network flows" Addison Wesley. Reading 1969.

[13] Mangarasian. "Linear and non-linear programming" Acad. Press. 1974.

[14] Farkas, J. "Uber die theorie der einfachem Ungleichmigen" J. Reine Argew. Math 124, pp.1-24, 1902.

[15] Marchi, E. "On the green wave in a rectangular city". Proceedings Symposium. ISATA. Florence, June pp. 16-19. 1997.

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