Summary

The (γ, n) cross section of Nb^{93} was measured by the residual activity method. Combining this result with the total neutrons yield curve it was possible to separate the contributions of the (γ, n) and $(\gamma, 2n)$ reactions.

The ratio of these cross sections was then compared with special attention to the 2.35 isomeric state $(T_{1/2} = 13$ hs) reported by James; this activity was not found and we determined an upper limit for it as 0.02 % of the 0.93 Mev line. An activity of 3,2 % abundance due probably to γ rays of annihilation of positrons was found.

ON THE PROBLEM OF CANONICAL FIELD QUANTIZATION

por HANS JOOS
(Facultade de Filosofia, Ciências e Letras. Departamento de Fisica (*). Sao Paulo, Brasil).

The application of canonical quantization to non-linear relativistic field theory involves difficulties, which are not yet clarified. We wish to discuss some points of this problem.

1. A classical field $\varphi_r(x)$ is described by a relativistic invariant Lagrange density $L(\varphi_r, \varphi_{r/x'})$ (1). From L we derive:

a) The *field equations* as Euler-Lagrange equations of the Hamilton principle $\delta \int L\, dx = 0$:

$$\frac{\partial}{\partial x^\mu} \frac{\partial L}{\partial \varphi_{r/x^\mu}} - \frac{\partial L}{\partial \varphi_r} = 0. \qquad (1)$$

(*) Contracted by the "Conselho Nacional das Pesquisas" of Brasil. Now at the "Institut für theoretische Physik der Universität Hamburg".

(1) We use the following notation: Space-time point $x = (x^\mu) = (x^0, \vec{x})$; invariant length $x^2 = x^\mu x_\mu = -\vec{x}^2 + x^{0\,2} = x^\mu g_{\mu\nu} x^\nu$ derivative $\frac{\partial}{\partial x^\mu} \varphi_r = \varphi_{r|x\mu}$; r denotes the spin and iso-spin indices etc.

b) The *canonical conjugate* of $\varphi_r(x)$:

$$\pi_r(x) = \frac{\partial L}{\partial \varphi_{r/x^0}} \cdot \tag{2}$$

c) The *generating functionals* of the infinitesimal transformations *of the inhomogeneous Lorentz group*:

$$P_\mu[\pi, \varphi; t] = \sum_r \int_{(x^0=t)} d\vec{x} \left(\pi_r(x) \frac{\partial}{\partial x^\mu} \varphi_r(x) - g_{0\mu} L\right)$$

for the infinitesimal translations

$$\delta \phi = \frac{\partial}{\partial x^\mu} \varphi \, \delta\tau, \tag{3}$$

$$M_{\mu\nu}[\pi, \varphi; t] = \sum_{r, r'} \int_{x^0=t} dx \, (\pi_r(\vec{x}) \left(\left(x_\mu \frac{\partial}{\partial x^\nu} - x_\nu \frac{\partial}{\partial x^\mu}\right) \delta^{rr'} + S_{\mu\nu}{}^{rr'}\right) \varphi_{r'}(x)$$

$$+ (x \, g_0 - g_0 \, x \, L)$$

for the infinitesimal rotations

$$\delta \varphi_r = \left(\left(x_\mu \frac{\partial}{\partial x^\nu} - x_\nu \frac{\partial}{\partial x^\mu}\right) \varphi_r + \sum_{r'} S_{\mu\nu}{}^{rr'} \varphi_{r'}\right) \delta\tau. \tag{3'}$$

P_μ has the physical meaning of the relativistic energy —momentum 4— vector, $M_{\mu\nu}$ of the relativistic angular momentum.

We define for functionals $F[\pi, \delta; t]$, with fixed t, Poisson brackets

$$\{F_1, F_2\} = \sum_r \int d\vec{x} \left(\frac{\delta F_1}{\delta \pi_r(x)} \frac{\delta F_2}{\delta \varphi_r(x)} - \frac{\delta F_1}{\delta \varphi_r(x)} \frac{\delta F_2}{\delta \pi_r(x)} \right). \tag{4}$$

The following set of Poisson bracket relations are direct consequences of (1) — (4):

a') The canonical form of the field equations

$$\{P_\mu, \ \varphi_r(x)\} = \frac{\partial}{\partial x^\mu} \varphi_r(x), \{P_\mu, \pi_r(x)\} = \frac{\partial}{\partial x^\mu} \varphi_r(x)$$

$$\{M_{\mu\nu}, \varphi_r(x)\} = \left(x_\mu \frac{\partial}{\partial x^\nu} - x_\nu \frac{\partial}{\partial x^\mu}\right) \varphi_r(x) + \sum_{r'} S_{\mu\nu}{}^{rr'} \varphi_{r'}(x). \tag{5}$$

b') The characteristic Poisson brackets of canonical conjugate field quantities

$$\{\varphi_r(x,t),\varphi_{r'}(x',t)\} = \{\pi_r(x,t),\pi_{r'}(x' t)\} = 0$$

$$\{\pi_r(x,t),\varphi_{r'}(x',t)\} = \delta_{rr'}\,\delta\,(\vec{x}-\vec{x}'). \qquad (6)$$

c') The Poisson brackets of relativistic 4-momentum and angular momentum, which contain the conservation laws of energy, momentum, angular momentum and the centre of mass

$$\{M_{\mu\nu}, M_{\rho\tau}\} = g_{\nu\rho}\,M_{\mu\tau} - g_{\mu\rho}\,M_{\nu\tau} + g_{\mu\tau}\,M_{\nu\rho} - g_{\nu\tau}\,M_{\mu\rho}$$

$$\{M_{\mu\nu}, P_\rho\} = g_{\nu\rho}\,P_\mu - g_{\mu\rho}\,P_\tau, \;, \; \{P_\mu, P_\nu\} = 0. \qquad (7)$$

In the scheme of canonical quantum field theory, $\varphi_r(x)$, $\pi_r(x)$ are substituted by hermitian operators $\varphi_r(x), \pi_r(x)$, which satisfy the canonical commutation relations CR:

$$\varphi_r(x,t)\,.\,\varphi_{r'}(x',t) - \varphi_{r'}(x',t)\,\varphi_r(x,t) = [\varphi_r(x,t),\varphi_{r'}(x',t)] = 0$$

$$[\pi_r(x,t),\pi_{r'}(x',t)] = 0; \quad i[\pi_r(x,t),\pi_{r'}(x',t)] = \delta_{rr'}\,\delta(x-x'). \qquad (6')$$

Substituting in the functionals $P_\nu, M_{\mu\nu}$ the $\varphi_r(x)$, $\pi_r(x)$ by the operators $\varphi_r(x)$, $\pi_r(x)$, we get operators P_ν, $M_{\mu\nu}$ satisfying the relations (5) - (7), if we interpretate $\{F_1, F_2\}$ as $i\,[F_1, F_2]$ (2). There is of course the wel known arbitrariness in the ordering of the operators.

For a finite number of canonical coordinates p_m, q_m, $m = 1 \dots n$ the C. R. possess only one type of inequivalent irreducible representations. In this case the canonical scheme determines uniquely the quantum mechanical properties of a physical system, i. e. the eigenvalues of the observables and the transition amplitudes.

2. There are different, not equivalent representations (3) of the C. R. in the case of a infinite number of degrees of freedom, i. e. in field theory. In order to discuss this fact, we choose as

(2) $\hbar = 1, c = 1$

(3) FRIEDRICHS, *Mathematical Aspects of Quantum Theory of Fields*, *Interscience publishers*, New York (1953).

L. VAN HOVE, *Physica*, Vol. 18 (1952) p. 145-159.

example a countable infinite number of canonical coordinates $p_m, q_m, m=1, \dots$ with the C. R.:

$$[p_m, q_{m'}] = -i\,\delta_{mm'}, \quad [p_m, p_{m'}] = [q_m, q_{m'}] = 0.$$

The case (6′) may be reduced to this problem by expanding $\varphi_r(x), \pi_r(x)$ in a complete orthonormal system.

We consider representations of the C. R. by operators, which act on a Hilbert space of functions $\phi[y_1, \dots y_m, \dots]$ of infinitly many variables y_m. It is assumed that the ϕ are square integrable with respect to the product measure (4)

$$\prod_m (\pi\,\omega_m)^{1/2} \cdot \exp(-\,\omega_m\, y_m^2)\, dy_m, \omega_m > 0:$$

$$\|\phi\|^2 = \lim_{m\to\infty} \int_{-\infty}^{+\infty} dy_1 \dots \int_{-\infty}^{+\infty} dy_m |\phi[y_1 \dots y_m, y_{m+1}, \dots]|^2 \prod_{s=1}^{m} \frac{\exp(-\omega_s\, y_s^2)}{(\omega_s \pi)^{1/2}}.$$

The existence of this infinite dimensional integral, the norm of ϕ, implies that $|\phi[y_1, \dots y_m, \dots]|$ does not depend strongly on the y_m with large m, for allmost all sequences (y_s). We may choose now as hermitian operators, which satisfy the C.R.:

$$\text{I.} \quad p_m = \frac{\partial}{i\partial y_m} + i\,\omega_m\, y_m, \quad q_m = y_m$$

$$\text{II.} \quad p_m{}' = (\omega_m/\omega_m{}')^{1/2} \left(\frac{\partial}{i\partial y_m} + i\,\omega_m\, y_m\right), \quad q_m{}' = (\omega_m{}'/\omega_m)^{1/2}\, y_m. \tag{9}$$

I. and II. define non-equivalent representations of the C. R. if $\omega_m \not\to \omega_m{}'$. This is shown by the following remark. In representation I the partial Hamiltonians

$$H_m(p_m, q_m) = 1/2\,(p_m^2 + q_m^2\, \omega_m^2)$$

for oscillations with frequency ω_m have a simultaneous eigenstate with the eigenvalues 0: $\phi[y] \equiv 1$.

In representation II such a simultaneous eigenstate does not exist, because the formal eigenstate

$$\prod_m (\omega_m/\omega_m{}')^{1/2} \exp((\omega_m - \omega_m{}') \cdot y_m^2)$$

(4) P. J. Daniell, *Annals of Math.* Vol. 20 (1919), p. 281-288.
B. Jessen, *Acta Math.* Vol. 63 (1934), p. 249-323.

is for almost all (y_s) 0 or ∞ according as

$$\omega_m' > \omega_m \quad \text{or} \quad \omega_m' < \omega_m.$$

This formal vector is therefore not an element of our Hilbert space with $\|\Phi\| \neq 0$.

Having in mind the separation of the linear field Hamiltonians in harmonic oscillators, we may draw the following conclusions:

a) There are non-equivalent representations of the C. R. A relativistic Hamiltonian operator, which is constructed according to the canonical scheme, can be defined as a self-adjoint operator only in a determined representation ([5]). In our example $H = \sum_m H_m$ is a self-adjoint operator in representation *I*, but H can not be defined in representation II.

b) In the quantum theory of free fields it is not posible to consider a state with a certain energy and momentum distribution of a particle with a certain rest mass as a superposition of many particle states of particles with different rest masses, without violating the Hilbert space formalism of quantum mechanics. This is the physical interpretation of statement a). A similar problem may appear in the decomposition of «real particles» in «free particles», which is used in the perturbation treatment of non-linear field theory.

c) Using the functional form for the description of the different representation of the C. R., we may get an appropriate representation with help of a positive definite functional, for which is formally

$$H\left(\frac{\delta\varphi}{i\delta\varphi_r(x)},\ \varphi_r(x)\right)\Phi = 0$$

and which defines a norm in the functional space similar to (8).

We may resume our discussion by the statement, that the existence of non-equivalent representations of the C. R. changes the problem of canonical quantization in field theory. Between the infinite variety of different representations one has to look for the special representation, for which the canonical operators of the physical important quantities exist. It is not yet decided,

([5]) R. HAAG, *Cern —report T/RH— 1.*

if such a representation exists for any relativistic non-linear field theory.

3. It is interesting to note, that the application of the operator method (6) to classical field mechanics involves similar questions as canonical field quantization.

Formally we may associate a linear operator $\Omega(F)$ to any generating functional $F[\Pi, \varphi]$ of an infinitesimal canonical transformation:

$$\Omega(F)\,\psi = \Big(F - \sum_r \int d\vec{x}\,\pi_r(\vec{x})\frac{\delta F}{\delta \pi_r(\vec{x})}\Big)\psi - i\{F, \psi\} \tag{10}$$

which acts upon functionals $\psi[\pi_r(\vec{x}), \varphi_r(\vec{x})]$.

Resembling canonical quantum mechanics, we get

$$i[\Omega(F_1),\ \Omega(F_2)] = \Omega(\{F_1, F_2\}). \tag{11}$$

Particularly the operators $\Omega(\pi_r(\vec{x}))$, $\Omega(\varphi_r(\vec{x}))$, $\Omega(P_\nu)$, $\Omega(M_{\mu\nu})$ satisfy commutation relations like (5) - (7).

Applications in statistical mechanics (7) require a generalization of Liouville's theorem for a infinite number of degrees of freedom. For this purpose we have to look for a measure in the infinite dimensional phase space of the canonical conjugate functions $\varphi_r(\vec{x})$, $\pi_r(\vec{x})$, which is invariant under the canonical transformations of the inhomogeneous Lorentz group. In order to show the similarity of this problem to those discussed in 2, we choose once more as an example a system with a countable number of degrees of freedom, and with the Hamiltonian $H = \sum_m 1/2\,(p_m{}^2 + \omega_m{}^2 q_m{}^2)$. According to (10) we get

$$\Omega(H) = \sum_m \Big(\frac{1}{2}(\omega_m{}^2 q_m{}^2 - p_m{}^2) - i\Big(p_m\frac{\partial}{\partial q_m} - \omega_m{}^2 q_m \frac{\partial}{\partial p_m}\Big)\Big) \tag{12}$$

$\Omega(H)$ is a self-adjoint operator in the Hilbert space of functionals $\psi[p_1, q_1, \ldots]$, which are square integrable with respec

(6) KOOPMANN, *Nac. Ac. Scie.* Vol. 17 (1931) p. 315.

L. VAN HOVE, *Académie royale de Belgique, Classe de Sciences* Tomé XXVI, Fasc. 6.

(7) J. KAMPÉ DE FÉRIET, *Proc. of the second Berkeley Symposium on Math. Statistics and Probability* (1951) p. 553.

to the product measure $\prod_m \exp(g_m(p^2+\omega_m{}^2 q^2))\, dp_m dq_m$. (The arbitrary functions $g(z)$ must be normalized: $\iint \exp(g_m(p_m{}^2+ + \omega_m{}^2 q_m{}^2)) \,.\, dp_m\, dq_m = 1.$). The eigenvectors of $\Omega(H)$ with the eigenvalues $E = \Sigma\, n_m\, \omega_m$ are $\psi_E = \prod_m (\exp(i/2\; p_m\, q_m)) \,.\, (p_m + \mathrm{sign}(n_m)\,.\,\omega_m\, q_m)^{|n_m|}$. In order ψ_E is square integrable only if a finite number of the n_m is allowed to be different from 0.

The treatment of classical linear fields along these lines is straightforward. The measure becomes unique by the postulate of relativistic invariance. The construction of an invariant measure in non-linear field theory involves problems similar to those discussed in 2c.

ON THE UNITARY REPRESENTATIONS OF THE LORENTZ GROUP

por Hans Joos

(Facultade de Filosofia, Ciências e Letras. Departamento de Fisica (*). Sao Paulo, Brasil).

In relativistic quantum mechanics the kinematical properties of closed systems are determined by the properties of the unitary representations of the inhomogeneous Lorentz group (iLG) and of its different sub-groups. In this paper we discuss the complete reduction of the representations of the homogeneous, proper Lorentz group (pLG), which are contained in the irreducible unitary representations of the iLG.

The pLG consists of the transformations of the space time coordinates $x^\mu, \mu = 0, 1, 2, 3,$

$$x^{\mu\prime} = \Lambda_\nu{}^\mu\, x^\nu \tag{1}$$

which conserve

$$x^2 = x^\mu x_\mu = x^\mu g_{\mu\nu} x^\nu = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = (x^0)^2 - (\vec{x})^2$$

and space and time orientation: $\det \Lambda = 1,\ \Lambda_0{}^0 > 0$. The iLG

(*) Contracted by the *Conselho Nacional das Pesquisas*, of Brasil. *Now at the Institut für theoretische Physik der Universität Hamburg.*