Revista de la
Unión Matemática Argentina
Volumen 24, Número 4, 1969.

SOME FORMULAE FOR G-FUNCTION

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1. INTRODUCTION. In a recent paper {5} the author has defined the generalized function of two variables as follow

$$(1)\quad S\left[\begin{array}{c}\left[\begin{matrix} m_1 , 0 \\ p_1-m_1 , q_1\end{matrix}\right] \\ \left(\begin{matrix} m_2 , n_2 \\ p_2-m_2 , q_2-n_2\end{matrix}\right) \\ \left(\begin{matrix} m_3 , n_3 \\ p_3-m_3 , q_3-n_3\end{matrix}\right)\end{array} \left| \begin{array}{c} \alpha_1,\dots,\alpha_{p_1} ; b_1,\dots,b_{q_1} \\ c_1,\dots,c_{p_2} ; d_1,\dots,d_{q_2} \\ e_1,\dots,e_{p_3} ; f_1,\dots,f_{q_3}\end{array}\right| \begin{array}{c} x, \\ y \end{array}\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{c_1}\int_{c_2} \frac{\prod_{j=1}^{m_1}\Gamma(\alpha_j+s+t)\prod_{j=1}^{m_2}\Gamma(1-c_j+s)\prod_{j=1}^{n_2}\Gamma(d_j-s)\prod_{j=1}^{m_3}\Gamma(1-e_j+t)}{\prod_{j=m_1+1}^{p_1}\Gamma(1-\alpha_j-s-t)\prod_{j=1}^{q_1}\Gamma(b_j+s+t)\prod_{j=m_2+1}^{p_2}\Gamma(c_j-s)\prod_{j=n_2+1}^{q_2}\Gamma(1-d_j+s)}$$

$$\frac{\prod_{j=1}^{n_3}\Gamma(f_j-t)}{\prod_{j=m_3+1}^{p_3}\Gamma(e_j-t)\prod_{j=n_3+1}^{q_3}\Gamma(1-f_j+t)}\; x^s\, y^t\, ds\, dt \quad .$$

where c_1 and c_2 are two suitable contours.

In other two papers {6,7} the author has discussed the simple properties and particular cases of the generalized function of two variables.

The object of this paper is to evaluate integrals involving Meijers' G-function in terms of generalized function of two variables.

The formulae obtained are of general character and in particular we obtain some interesting integrals. The results are believed to be new.

In the investigation we require the formula {8}

$$(2)\quad \int_0^\infty x^{\lambda-i} e^{-px}\, G^{n_2,m_2}_{p_2,q_2}\left[\alpha x^n \,\middle|\, \begin{matrix} c_1,\dots,c_{p_2} \\ d_1,\dots,d_{q_2} \end{matrix}\right] G^{n_3,m_3}_{p_3,q_3}\left[b x^n \,\middle|\, \begin{matrix} e_1,\dots,e_{p_3} \\ f_1,\dots,f_{q_3} \end{matrix}\right] dx =$$

$$\frac{(2\pi)^{\frac{1}{2}(1-n)}\,(n)^{\lambda-\frac{1}{2}}}{p^\lambda}\, S\left[\begin{array}{c|l|c} \begin{bmatrix} n,o \\ o,o \end{bmatrix} & \phi_1,\dots,\phi_n\;; & \alpha\left(\frac{n}{p}\right)^n \\ \begin{pmatrix} m_2,n_2 \\ p_2-m_2,q_2-n_2 \end{pmatrix} & c_1,\dots,c_{p_2};d_1,\dots,d_{q_2} & \\ \begin{pmatrix} m_3,n_3 \\ p_3-m_3,q_3-n_3 \end{pmatrix} & e_1,\dots,e_{p_3};f_1,\dots,f_{q_3} & b\left(\frac{n}{p}\right)^n, \end{array}\right]$$

where $\phi_{k+1} = \frac{\lambda+k}{n}$, $k=0,1,\dots,\overline{n-1}$, n is a positive integer, $2(m_2+n_2) > p_2+q_2$, $2(m_3+n_3) > p_3+q_3$, $|\arg \alpha| < (m_2+n_2-\frac{1}{2}q_2-\frac{1}{2}p_2)\pi$, $|\arg b| < (m_3+n_3-\frac{1}{2}p_3-\frac{1}{2}q_3)\pi$, $R(p) > 0$, $R(\lambda+nd_y+nf_h) > 0$, $y=1,2,\dots,n_2$; $h = 1,2,\dots,n_3$.

We write, as usual $\phi(p) \doteqdot h(t)$ when

$$(3)\qquad \phi(p) = p\int_0^\infty e^{-pt}\, h(t)\, dt\,.$$

INTEGRALS.

2. The first formula to be proved is

$$(4)\quad \int_0^\infty t^{-\rho}(t+c)^{\lambda-1}\, G^{n_2,m_2}_{p_2,q_2}\left[\frac{\alpha}{(t+c)^n} \,\middle|\, \begin{matrix} \alpha_1,\dots,\alpha_{p_2} \\ b_1,\dots,b_{q_2} \end{matrix}\right] G^{n_3,m_3}_{p_3,q_3}\left[\frac{b}{t^n} \,\middle|\, \begin{matrix} c_1,\dots,c_{p_3} \\ d_1,\dots,d_{q_3} \end{matrix}\right] dt$$

$$= \frac{(2\pi)^{\frac{1}{2}(1-n)}}{\sqrt{n}\,(c)^{\rho-\lambda}}\, S\left[\begin{array}{c|l|c} \begin{bmatrix} n,o \\ o,o \end{bmatrix} & \delta_1,\dots,\delta_n\;; & \frac{\alpha}{c^n}, \\ \begin{pmatrix} m_2,n_2 \\ p_2-m_2,q_2-n_2+n \end{pmatrix} & \alpha_1,\dots,\alpha_{p_2};\; b_1,\dots,b_{q_2},\phi_1,\dots,\phi_n & \\ \begin{pmatrix} m_3,n_3+n \\ p_3-m_3,q_3-n_3 \end{pmatrix} & c_1,\dots,c_{p_3};\psi_1,\dots,\psi_n,d_1,\dots,d_{q_3} & \frac{b}{c^n} \end{array}\right]$$

where n is a positive integer, $\delta_{k+1} = \frac{\rho-\lambda+k}{n}$, $\phi_{k+1} = \frac{\lambda+k}{n}$, $\psi_{k+1} =$ $= \frac{1-\rho+k}{n}$, $k = 0,1,\ldots,\overline{n-1}$, $2(m_2+n_2) > p_2+q_2$, $|\arg \alpha| <$ $< (m_2+n_2-\frac{1}{2}p_2-\frac{1}{2}q_2)\pi$, $2(m_3+n_3) > p_3+q_3$, $|\arg b| <$ $< (m_3+n_3-\frac{1}{2}p_3-\frac{1}{2}q_3)\pi$, $|\arg c| < \pi$, $R(\rho+nc_y) < n+1$, , $y=1,2,\ldots,m_3$, $R(\rho-\lambda+nb_y+nd_h) > 0$, $y = 1,2,\ldots,n_2$; $h = 1,2,\ldots,n_3$.

Proof: In the proof we use the Parseval-Goldstien {3} theorem of operational calculus; that is

$$\phi(p) \doteqdot h(t) \quad \text{and} \quad \psi(p) \doteqdot g(t)$$

then

$$\int_0^\infty t^{-1}\,\phi(t)\,g(t)dt = \int_0^\infty t^{-1}\,h(t)\,\psi(t)\,dt \qquad (5)$$

where the integrals are convergents.

Now we take {4, p. 402 equ. 11}

$$h(t) = t^{-\lambda} e^{-ct}\, G^{n_2,m_2}_{p_2,q_2+n}\left[\frac{\alpha t^n}{(n)^n}\,\middle|\,\begin{matrix}\alpha_1,\ldots,\alpha_{p_2} \\ b_1,\ldots,b_{q_2},\ \phi_1,\ldots,\phi_n\end{matrix}\right] \qquad (6)$$

$$\doteqdot (2\pi)^{\frac{1}{2}(1-n)}\,(n)^{\frac{1}{2}-\lambda}\,p(p+c)^{\lambda-1}\, G^{n_2,m_2}_{p_2,q_2}\left[\frac{\alpha}{(p+c)^n}\,\middle|\,\begin{matrix}\alpha_1,\ldots,\alpha_{p_2} \\ b_1,\ldots,b_{q_2}\end{matrix}\right] = \phi(p) ,$$

where n is a positive integer, $\phi_{k+1} = \frac{\lambda+k}{n}$, $k = 0,1,\ldots,\overline{n-1}$, $2(m_2+n_2) > p_2+q_2+n$, $|\arg \alpha| < (m_2+n_2-\frac{1}{2}p_2-\frac{1}{2}q_2-\frac{1}{2}n)\pi$, $R(1-\lambda+nb_y) > 0$ $y = 1,2,\ldots,n_2$; $R(p+c) > 0$.

By using the formula {1, p.209, Equ. 9} , we have from {4,p.402 Equ. 11}

$$g(t) = t^{-\rho}\, G^{n_3,m_3}_{p_3,q_3}\left[\frac{b}{t^n}\,\middle|\,\begin{matrix}c_1,\ldots,c_{p_3} \\ d_1,\ldots,d_{q_3}\end{matrix}\right] \qquad (7)$$

$$+ (2\pi)^{\frac{1}{2}(1-n)} (n)^{\frac{1}{2}-\rho} p^{\rho} G^{n_3+n,m_3}_{p_3,q_3+n}\left[\frac{bp^n}{(n)^n}\middle| \begin{matrix} c_1,\ldots,c_{p_3} \\ \psi_1,\ldots,\psi_n , d_1,\ldots,d_{q_3}\end{matrix}\right] = \psi(p),$$

where n is a positive integer, $\psi_{k+1} = \frac{1-\rho+k}{n}$, $k = 0,1,\ldots,\overline{n-1}, R(p) > 0$, $2(n_3+m_3) > p_3+q_3$, $|\arg b| < (m_3+n_3-\frac{1}{2} p_3 - \frac{1}{2} q_3)\pi$, $R(\rho+nc_j) < n+1$, $j = 1,2,\ldots,m_3$.

Using (6) and (7) in (5), we get

$$(8) \int_0^\infty t^{-\rho}(t+c)^{\lambda-1} G^{n_2,m_2}_{p_2,q_2}\left[\frac{\alpha}{(n)^n}\middle| \begin{matrix} \alpha_1,\ldots,\alpha_{p_2} \\ b_1,\ldots,b_{q_2}\end{matrix}\right] G^{n_3,m_3}_{p_3,q_3}\left[\frac{b}{t^n}\middle| \begin{matrix} c_1,\ldots,c_{p_3} \\ d_1,\ldots,d_{q_3}\end{matrix}\right] dt =$$

$$= (n)^{\lambda-\rho}$$

$$\int_0^\infty t^{\rho-\lambda-1} e^{-ct} G^{n_2,m_2}_{p_2,q_2+n}\left[\frac{\alpha t^n}{(n)^n}\middle| \begin{matrix} \alpha_1,\ldots,\alpha_{p_2} \\ b_1,\ldots,b_{q_2},\phi_1,\ldots,\phi_n\end{matrix}\right] G^{n_3+n,m_3}_{p_3,q_3+n}\left[\frac{bt^n}{(n)^n}\middle| \begin{matrix} c_1,\ldots,c_{p_3} \\ \psi_1,\ldots,\psi_n , d_1,\ldots,d_{q_3}\end{matrix}\right] dt$$

(4) follows immediately on evaluating the integral on the right of (8) with the help of (2).

PARTICULAR CASES. We shall mention below some interesting particular cases of our general result (4)

(a) Taking $n = 1$, $m_2 = 2$, $n_2 = 2$, $p_2 = 2$, $q_2 = 2$, $n_3 = 2$, $m_3 = 1$, $p_3 = 2$, $q_3 = 2$ and using the formula (7)

$$(9)\ S\left[\begin{matrix}\begin{bmatrix}1,0\\0,0\end{bmatrix}\\ \begin{pmatrix}1,2\\0,0\end{pmatrix}\\ \begin{pmatrix}1,2\\0,0\end{pmatrix}\end{matrix}\middle|\begin{matrix}\lambda; \\ 1-\alpha;\delta,-\delta \\ 1-\beta;\rho,-\rho\end{matrix}\middle|\begin{matrix}x, \\ \\ y\end{matrix}\right] = \sum_{\delta,-\delta}\ \sum_{\rho,-\rho} \Gamma(\lambda+\rho+\delta)\Gamma(\alpha+\delta)\Gamma(\beta+\rho)\Gamma(-2\rho)\Gamma(-2\delta)$$

$x^{\delta}\, y^{\rho}\quad F_2\,[\,\lambda+\delta+\rho\ ;\ \alpha+\delta\ ,\ \beta+\rho\ ;\ 1+2\delta\ ,\ 1+2\rho\ ;\ x\ ,\ y\,]$

in (4), we get

$$(10)\quad \int_0^{\infty} t^{\gamma-\rho}\ (t+c)^{\lambda-1}\ {}_2F_1(\alpha+\gamma,\gamma-\alpha;1+\gamma-\rho;-\tfrac{t}{b})\ G^{22}_{22}\left[\frac{t+c}{\alpha}\ \middle|\ \begin{matrix}1-\delta,1+\delta\\ 1-\lambda,\ \beta\end{matrix}\right] dt =$$

$$= \frac{b^{\gamma}\ \Gamma(1+\gamma-\rho)}{\Gamma(\gamma\pm\alpha)} \sum_{\delta,-\delta}\ \sum_{\alpha,-\alpha} \Gamma(\rho-\lambda+\alpha+\delta)\Gamma(\beta+\delta)\Gamma(\gamma+\alpha)\Gamma(-2\alpha)\Gamma(-2\delta)\alpha^{\delta}b^{\alpha}c^{\lambda-\rho-\alpha-\delta}$$

$$F_2\,[\,\rho-\lambda+\alpha+\delta\ ;\ \beta+\delta,\gamma+\alpha\ ;\ 1+2\delta,1+2\alpha\ ;\ \tfrac{\alpha}{c}\ ,\ \tfrac{b}{c}\,]\qquad ,$$

valid for $R(\gamma-\rho+1) > 0$, $R(\lambda-\rho\pm\alpha\pm\delta) < 0$, $R(c) > R(\alpha)$, $R(c) > R(b)$, $|\arg c| < \pi$.

(b) Taking $n = 1$, $m_2 = 1$, $n_2 = 2$, $q_2 = 2$, $p_2 = 1$, $n_3 = 2$, $m_3 = 0$, $p_3 = 1$, $q_3 = 2$, using the formulae {1, p.216 Equ.6. Equ.8} and (7)

$$(11)\quad S\left[\begin{array}{c|c|c} \begin{bmatrix}1,0\\0,0\end{bmatrix} & \rho; & \\ \begin{pmatrix}0,2\\0,0\end{pmatrix} & ;\alpha,-\alpha & x, \\ \begin{pmatrix}0,2\\0,0\end{pmatrix} & ;\beta,-\beta & y \end{array}\right] = \sum_{\alpha,-\alpha}\ \sum_{\beta,-\beta} \Gamma(\rho+\alpha+\beta)\Gamma(-2\alpha)\Gamma(-2\beta)\ x^{\alpha}\ y^{\beta}$$

$\psi_2\ [\ \rho+\alpha+\beta\ ;\ 1+2\alpha,1+2\beta;\ x,y]$

in (4),we get

$$(12)\quad \int_0^{\infty} t^{-\rho}\ (t+c)^{\lambda}\ \exp\left[\frac{\alpha}{2(t+c)} - \frac{b}{2t}\right] W_{\lambda,\alpha}\left[\frac{b}{t+c}\right] W_{\rho,\beta}\left[\frac{b}{t}\right] dt =$$

$$= \frac{1}{\Gamma(1\pm2\alpha)} \sum_{\alpha,-\alpha}\ \sum_{\beta,-\beta} \frac{(a)^{\alpha+\frac{1}{2}}\ (b)^{\beta+\frac{1}{2}}}{(c)^{\rho-\lambda+\alpha+\beta}}\ \Gamma(\rho-\lambda+\alpha+\beta)\ \Gamma(-2\alpha)\ \Gamma(-2\beta)$$

$$\psi_2\ [\ \rho-\lambda+\alpha+\beta\ ;\ 1+2\alpha,1+2\beta\ ;\ \tfrac{\alpha}{c}\ ,\ \tfrac{b}{c}\ ]\qquad ,$$

valid for $R(b) > 0$, $R(c) > R(\alpha)$, $R(\rho-\lambda\pm\alpha\pm\beta) > 0$.

(c) Taking $n = 2$, $m_2 = 2$, $n_2 = 1$, $p_2 = 2$, $q_2 = 2$, $m_3 = 0$, $n_3 = 1$, $q_3 = 2$, $p_3 = 2$ and using the formula (7)

$$(13)\quad S\left[\begin{matrix}\begin{bmatrix}2,0\\0,0\end{bmatrix}\\ \begin{pmatrix}0,1\\0,1\end{pmatrix}\\ \begin{pmatrix}0,1\\0,1\end{pmatrix}\end{matrix}\left|\begin{matrix}\lambda,\mu\;;\\ \;;\;0,1-\alpha\\ \;;\;0,1-\beta\end{matrix}\right|\begin{matrix}x,\\ y\end{matrix}\right] = \frac{\Gamma(\lambda)\,\Gamma(\mu)}{\Gamma(\alpha)\,\Gamma(\beta)}\times$$

$$F_4\left[\lambda,\mu\;;\;\alpha,\beta\;;\;-x,-y\right]$$

in (4), we get

$$(14)\int_0^\infty t^{-\rho}\,(t+c)^{\lambda-1}\;{}_2F_1\left[\tfrac{1}{2}(2-\lambda),\,\tfrac{1}{2}(1-\lambda)\;;\;\alpha\;;\;-\frac{a}{(t+c)^2}\right]$$

$$G_{22}^{01}\left[\frac{t^2}{b}\left|\begin{matrix}1\;,\;\beta\\ \tfrac{1}{2}(1+\rho),\;\tfrac{1}{2}\rho\end{matrix}\right.\right]dt = \frac{(c)^{\lambda-\rho}\,\Gamma(\rho-\lambda/2)\,\Gamma(\rho-\lambda+1/2)}{\sqrt{4\pi}\,\Gamma(\beta)\,\Gamma(1-\frac{1}{2}\lambda)\,\Gamma(\frac{1}{2}-\frac{1}{2}\lambda)}$$

$$F_4\left[\tfrac{1}{2}(\rho-\lambda),\,\tfrac{1}{2}(\rho-\lambda+1)\;;\;\alpha,\beta\;;\;-\frac{a}{c^2},-\frac{b}{c^2}\right],$$

valid for $R(\rho-\lambda)>0$, $R(c)>R(\sqrt{a})$, $R(c)>R(\sqrt{b})$, $R(c)>0$.

3. The second formula to be proved is

$$(15)\int_0^\infty t^{-\lambda}\,(a+bt+ct^2)^{\rho-1}\,G_{p_2,q_2}^{n_2,m_2}\left[(a+bt+ct^2)^{-1}\left|\begin{matrix}\alpha_1,\ldots,\alpha_{p_2}\\ b_1,\ldots,b_{q_2}\end{matrix}\right.\right]dt =$$

$$\frac{b^{\lambda-1}\,a^{\rho-\lambda}}{\sqrt{\pi}\,2^{\lambda}}S\left[\begin{matrix}\begin{bmatrix}1,0\\0,0\end{bmatrix}\\ \begin{pmatrix}m_2,n_2\\ p_2-m_2,q_2-n_2+1\end{pmatrix}\\ \begin{pmatrix}1,2\\0,0\end{pmatrix}\end{matrix}\left|\begin{matrix}\lambda-\rho\;;\\ \alpha_1,\ldots,\alpha_{p_2}\;;\;b_1,\ldots,b_{q_2}\;,\;\rho\\ 1\;;\;1-\tfrac{1}{2}\lambda\,,\,\tfrac{1}{2}-\tfrac{1}{2}\lambda\end{matrix}\right|\begin{matrix}\frac{1}{a}\,,\\ \frac{b^2}{4ac}\end{matrix}\right]$$

valid by analytic continuation, for $2(m_2+n_2)>p_2+q_2$, $R(1-\lambda)>0$, $R(\lambda-2\rho+2b_j+1)>0$, $R(c)>0$, $R(b)>0$, $R(a)>0$. $j=1,2,\ldots,n_2$.

Proof: In the proof, we use the theorem recently proved by the author {9} .

If $\phi(p) \doteqdot h(t)$

and $\psi(\lambda,p,\frac{b^2}{4c}) \doteqdot t^{\lambda-1}\ E(1-\frac{1}{2}\lambda,\frac{1}{2}-\frac{1}{2}\lambda :: \frac{b^2t}{4c})\ \ h(t)$

then

$$(16)\quad \int_0^\infty t^{-\lambda}\ (\alpha+bt+ct^2)^{-1}\ \phi(\alpha+bt+ct^2)\ dt = \frac{b^{\lambda-1}}{\alpha\sqrt{\pi}\,2^\lambda}\ \psi(\lambda,\alpha,\frac{b^2}{4c}) ,$$

provided that the integrals are absolutely convergent, $R(\alpha) > 0$, $|\arg \frac{b}{c}| < \frac{\pi}{2}$, $h(t)$ is independent of α.

We take {2,p. 222. Equ. 34}

$$(17)\quad h(t) = t^{-\rho}\ G^{n_2,m_2}_{p_2,q_2+1}\left[t \,\middle|\, \begin{matrix} \alpha_1,\ldots,\alpha_{p_2} \\ b_1,\ldots,b_{q_2}\ \ \rho \end{matrix} \right]$$

$$\doteqdot p^{\rho}\ \ G^{n_2,m_2}_{p_2,q_2}\left[\frac{1}{p} \,\middle|\, \begin{matrix} \alpha_1,\ldots,\alpha_{p_2} \\ b_1,\ldots,b_{q_2} \end{matrix} \right] = \phi(p) ,$$

valid for $R(p) > 0$, $R(1-\rho+b_y) > 0$, $y = 1,2,\ldots,n_2$; $2(p_2+q_2) > m_2+n_2+1$.

Also from (2), we have

$$(18)\quad \psi(\lambda,p,\frac{b^2}{4c}) = S\left[\begin{matrix} \begin{bmatrix} 1 , 0 \\ 0 , 0 \end{bmatrix} \\ \begin{pmatrix} m_2\ ,\ n_2 \\ p_2-m_2, q_2-n_2+1 \end{pmatrix} \\ \begin{pmatrix} 1\ ,\ 2 \\ 0\ ,\ 0 \end{pmatrix} \end{matrix} \,\middle|\, \begin{matrix} \lambda-\rho\ ; \\ \alpha_1,\ldots,\alpha_{p_2};b_1,\ldots,b_{q_2},\rho \\ 1\ ;1-\frac{1}{2}\lambda,\frac{1}{2}-\frac{1}{2}\lambda \end{matrix} \,\middle|\, \begin{matrix} \frac{1}{p} , \\ \\ \frac{b^2}{4cp} \end{matrix} \right]$$

By using (17) and (18) in (16), we get (15).

PARTICULAR CASES. We shall mention some interesting particular cases of the general result (15).

(a) Taking $n_2 = 1$, $q_2 = 1$, $m_2 = 0$, $p_2 = 0$ in (15), we get

$$(19) \int_0^\infty t^{-2\lambda} (\alpha+bt+ct^2)^{\rho-1} \exp\left[-\frac{1}{\alpha+bt+ct^2}\right] dt =$$

$$= \frac{(\alpha)^{\rho-\lambda-\frac{1}{2}} \Gamma(\lambda-\rho+\frac{1}{2}) \Gamma(\frac{1}{2}-\lambda)}{2(c)^{\frac{1}{2}-\lambda} \Gamma(1-\rho)} \psi_1\left[\lambda-\rho+\frac{1}{2}, \frac{1}{2}-\lambda ; \frac{1}{2}, 1-\rho ; \frac{b^2}{4\alpha c}, -\frac{1}{\alpha}\right] -$$

$$- \frac{b(\alpha)^{\rho-\lambda-1} \Gamma(\lambda-\rho+1) \Gamma(1-\lambda)}{2(c)^{1-\lambda} \Gamma(1-\rho)} \psi_1\left[\lambda-\rho+1, 1-\lambda ; \frac{3}{2}, 1-\rho ; \frac{b^2}{4\alpha c}, -\frac{1}{\alpha}\right] ,$$

valid for $R(\frac{1}{2}-\lambda) > 0$, $R(\frac{1}{2}+\lambda-\rho) > 0$, $R(\alpha) > 0$, $R(\alpha c) > R(\frac{b^2}{4})$.

In case $b \to 0$, (20) reduces to a known result {1,p.225. Equ. 2} .

For the definition of ψ_1 , see {1,p.225. Equ. 23} .

(b) Taking $n_2=1$, $m_2=1$, $p_2=1$, $q_2=1$, in (15), we get after a little simplification

$$(20) \int_0^\infty t^{-2\lambda} (\alpha+bt+ct^2)^{\rho+\alpha-1} (\alpha+1+bt+ct^2)^{-\alpha} dt =$$

$$= \frac{(\alpha)^{\rho-\lambda-\frac{1}{2}} \Gamma(\frac{1}{2}-\lambda) \Gamma(\lambda-\rho+\frac{1}{2})}{2(c)^{\frac{1}{2}-\lambda} \Gamma(1-\rho)} F_2\left[\lambda-\rho+\frac{1}{2}, \alpha, \frac{1}{2}-\lambda ; 1-\rho, \frac{1}{2} ; -\frac{1}{\alpha}, \frac{b^2}{4\alpha c}\right] -$$

$$- \frac{b(\alpha)^{\rho-\lambda-1} \Gamma(\lambda-\rho+1) \Gamma(1-\lambda)}{2(c)^{1-\lambda} \Gamma(1-\rho)} F_2\left[\lambda-\rho+1, \alpha, 1-\lambda ; 1-\rho, \frac{3}{2} ; -\frac{1}{\alpha}, \frac{b^2}{4\alpha c}\right] ,$$

Valid for $R(\frac{1}{2}-\lambda) > 0$, $R(\lambda-\rho+\frac{1}{2}) > 0$, $R(\alpha) > 1$, $R(\alpha c) > R(\frac{b^2}{4})$.

In case $b \to 0$, we get a known result {1, p.60 . Equ. 12} .

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