Revista de la
Unión Matemática Argentina
Volumen 25, 1970.

IDEALS AND UNIVERSAL REPRESENTANTIONS OF CERTAIN C*-ALGEBRAS

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INTRODUCTION. Let H be a Hilbert space and B(H) the C*-algebra of all bounded linear operators T: H → H . Consider the following two problems:

A) Find all the closed two-sided ideals of B(H).

B) For each closed two sided ideal J ⊂ B(H), find all the representations of the algebra B(H)/J in some Hilbert space.

The first problem has been solved by B.Gramsch [11], [12] (see also E. Luft [16]). The second has not yet been solved, even for the case J = {0}. See, however, [17, §22], [18], [20].

The solutions to A) obtained by Gramsch and Luft are based on a generalization of compactness: the ideals being characterized in terms of the "degree of compactness" shared by their constituent operators. The technique involves lengthy topological arguments . We remark that such generalizations of compactness abound: [21] , [8], [9] , [14] , [23] , [12]. In this note we describe an alternate approach to problem A) having only traditional notions of Hilbert space (projections, rank, etc.) as main ingredients, thereby avoiding generalized compactness. The arguments are considerably shortened.

Concerning problem B), except when J is the only maximal ideal of B(H), the dimension of the universal representation of B(H)/J is found to be equal to 2^{2^d} , where d = dim H . Observe that when J is the ideal of compact operators, there are faithful representations of B(H)/J of smaller dimension [20]. This is probably true for all non-maximal J.

§1. PRELIMINARIES AND NOTATION.

We shall observe the standard terminology for Hilbert spaces, as

Supported by NSF Grant GP 20431.

used in [6]. All throughout, H will denote a fixed complex Hilber space of dimension dim H = d = $\aleph_\delta$, where δ is some ordinal number $\delta = 0,1,\ldots$ etc. B(H) will denote the C*-algebra of all bounded linear operators T: H $\rightarrow$ H , C(H) and F(H) the two-sided ideals of B(H) of all compact operators and operators with finite rank, respectively. C(H) is closed in B(H) and F(H) dense in C(H). Greek letters α, β, γ will denote ordinal numbers in the interval $[0, \delta+1]$, so that $\aleph_0 \leqslant \aleph_\alpha \leqslant \aleph_{\delta+1}$. No misunderstanding should arise from a second use of β, in §3, to denote the Stone-Čech compactification βX of a topological space X. Let $\alpha \in [0,\delta]$, K Hilbert space of dimension $\aleph_\alpha$; we will denote by $m_\alpha \subset B(H)$ the set $m_\alpha = \{T\}$ of all operators of the form T = QS, where S: H $\rightarrow$ K , Q: K $\rightarrow$ H are linear and bounded. Obviously $\alpha \leqslant \beta$ implies $m_\alpha \subset m_\beta$. According to [19] or [1,§5], m_α is a two sided ideal of B(H), and if P $\in$ B(H) is a projection (= idempotent operator) with rank P=$\aleph_\alpha$; then ([19,1.3] or [1,5.14]), $m_\alpha = \{TPT' \; ; \; T,T' \in B(H)\}$. It follows from this characterization that $T \in m_\alpha$ if and only if the closed subspace generated by TH = {Tx ; x $\in$ H} has dimension at most $\aleph_\alpha$, and this at once implies that all m_α are norm closed in B(H) (in fact, they are also sequencially closed in the strong topology [5,§3, N° 1] of B(H)). If S is a set, Card S denotes the cardinal power of S. We assume the generalized continuum hypothesis , $(2^{\aleph_\alpha} = \aleph_{\alpha+1})$ although it is not used until 2.7.

§2. IDEALS OF B(H).

Let J be a two sided ideal of B(H). It is well-known that J is generated by the projections in J ([5, Chap. 1, §1, Ex. 6] , [4] , [25]). Actually, the same proof gives a better result:

2.1. LEMMA. *Let* J $\subset$ B(H) *be a two sided ideal and* T $\in$ J; *then* T *can be approximated (in norm) by operators of the form* TP, *where* P *is a hermitian projection and* P $\in$ J.

Proof. (cf. [*16, Lemma 5.2*] *)* : Set S = T*T and let $S = \int_0^\infty \lambda \, dP_\lambda$ be the spectral decomposition of $S \geqslant 0$. For $\varepsilon > 0$, define $K \subset H$ by $K = P_\varepsilon H$ and let $K^\perp$ denote the orthogonal subspace. Then :
a) K reduces S and $\| S|_K \| \leqslant \varepsilon$, b) for $x \in K^\perp$, $(Sx,x) \geqslant \varepsilon(x,x)$. It follows from b) that $\|Tx\| \geqslant \varepsilon^{1/2}\|x\|$ for all $x \in K^\perp$ and therefore

$TK^{\perp}$ is closed and $T: K^{\perp} \to TK^{\perp}$ is invertible. Let $L \in B(H)$ satisfy LT = identity on $K^{\perp}$. Then, if P denotes the orthogonal projection on K, we have $LTP = P$, whence $P \in J$, and $\|T-TP\|^2 = \|T|_K\|^2 = \|S|_K\| \leq \varepsilon$ the lemma follows.

2.2. REMARK. Assume that H is separable, and let $J \subset B(H)$ be a two sided ideal. If J contains a projection of infinite rank, then J contains also the identity $I: H \to H$, and therefore $J=B(H)$. On the other hand, if all projections in J have finite rank, then, by Lemma 2.1, J is contained in C(H). This shows an old result due to J.W. Calkin [2] : *C(H) is the largest proper two sided ideal of* B(H) (cf. [17, Chap. IV, §22, N° 1]).

Consider a two-sided ideal $J \subset B(H)$. We will associate to J an ordinal number h(J) and a two-sided ideal *J with some properties. First. the set of ordinals $\{\alpha \in [0,\delta] \; ; \; m_\alpha \subset J\}$, if not empty, is an initial segment, and therefore an ordinal h(J) is well determined by the properties a) $-1 \leq h(J) \leq \delta+1$; b) $h(J) = -1$ if and only if $J = \{0\}$; c) for $J \neq \{0\}$, $m_\alpha \subset J$ if and only if $\alpha < h(J)$. If $J = \{0\}$, set $*J = \{0\}$; if $J \neq \{0\}$, and $h(J) = 0$, set $*J = F(H)$; finally, if $h(J) > 0$, set $*J = \cup\{m_\alpha ; \alpha < h(J)\}$. It is clear that for all J, *J is also a two-sided ideal and $J \to h(J)$ and $J \to *J$ are monotonic: $J \subset K$ implies $h(J) \leq h(K)$ and $*J \subset *K$. Also, it is easy to see that $h(m_\alpha) = \alpha+1$.

2.3. LEMMA. *For all J we have* $*J \subset J \subset \overline{*J}$.

Proof. If $h(J) = 0$, then (Remark 2.2) $J \subset C(H)$ and therefore $\overline{*J} = \overline{F(H)} = C(H) \supset J$. Assume $h(J) > 0$ and take $T \in J$; according to Lemma 2.1, there is a projection $P \in J$ with $\|T-TP\| < \varepsilon$, for prescribed $\varepsilon > 0$. Clearly $m_\alpha = \{TPT' \; ; \; T,T' \in B(H)\} \subset J$, if rank $P = \aleph_\alpha$, and therefore $T \in \overline{*J}$. Thus $J \subset \overline{*J}$, as desired.

2.4. THEOREM ([11] , [16]). *The family $\mathcal{L}$ of all closed two-sided ideals in* B(H) *is well ordered by set inclusion. An ideal in $\mathcal{L}$ is of the form m_α if and only if it has an immediate predecessor in $\mathcal{L}$, different from* {0}.

Proof. We will show that the mapping $h: \mathcal{L} \to [-1,\delta+1]$ defined by $h: J \to h(J)$ is an order isomorphism onto $[-1,\delta+1]$. First, Lemma 2.3 shows that h is one-to-one on closed ideals: $h(J) = h(K)$ implies $*J = *K$ and therefore $J = \overline{*J} = \overline{*K} = K$. It was already observed that h is monotonic: $J \subset K$ implies $h(J) \leqslant h(K)$. The converse also holds: $h(J) \leqslant h(K)$ implies $*J \subset *K$, whence $J = \overline{*J} \subset \overline{*K} \subset K$. We show now that h is onto. Let $\beta \in [-1, \delta+1]$ and define $J_1 = \cup \{m_\alpha ; \alpha < \beta\}$, $J = \bar{J}_1$. Clearly $*J \subset J_1$ and therefore $h(J) \geqslant \beta$. Let P_β be a projection of rank $\aleph_\beta$. Then $\|P_\beta - T\| \geqslant 1$ for all $T \in m_\alpha$, for any $\alpha < \beta$. This is a general fact about closed ideals; if P is an idempotent and P does not belong to a closed two sided ideal K, then $\|P-T\| \geqslant 1$ for all $T \in K$. The proof is as follows: if $\|P-T\| = a < 1$, then $\|(P-T)^n\| \leqslant a^n \to 0$ and $(P-T)^n = P - S_n$, with $S_n \in K$. Thus $S_n \to P$ and $P \notin K$, a contradiction. Hence P_β does not belong to the closure of J_1, that is, to J and therefore $m_\beta \not\subset J$. Hence $h(J) \leqslant \beta$, and so $h(J) = \beta$, proving that h(J) covers $[-1, \delta+1]$. Finally, assume J has a predecessor $K \subset J$. Then $h(K) = \alpha$ and $h(J) = \alpha+1$ for some α. But also $h(m_\alpha) = \alpha+1$, so that by uniqueness $h(J) = h(m_\alpha)$ implies $J = m_\alpha$, as desired.

It is clear that the closed two-sided ideals of B(H) can be identified by their h(J), so that we may write J_α to denote the ideal J satisfying $h(J) = \alpha$. According to the proof of 2.4, we have J_α = closure $\cup\{m_\beta ; \beta < \alpha\}$. Then Lemma 2.1 can be reworded as follows:

2.5. $T \in J_\alpha$ *if and only if there is a sequence of commuting hermitian projections* $\{P_n\}$ *such that* $T = \lim TP_n$, *and* rank $P_n < \aleph_\alpha$ *for all* n.

We shall prove also the following generalization of Rellich criterion, due to E. Luft [16, Th. 5.2] :

2.6. $T \in J_\alpha$ *if and only if for each* $\varepsilon > 0$ *there is a subspace* $H_\varepsilon \subset H$ *with* codim $H_\varepsilon < \aleph_\alpha$ *such that* $\|T|_{H_\varepsilon}\| < \varepsilon$.

Proof. Assume this condition is satisfied, and let P_n be the orthogonal projection with nullspace H_ε for $\varepsilon = \frac{1}{n}$. Then rank $P_n < \aleph_\alpha$ and $\|T - TP_n\| = \|T(I-P_n)\| = \|T|_{H_\varepsilon}\| \leqslant \varepsilon$, so that 2.5 applies and $T \in J_\alpha$. The converse follows again from 2.5 taking $H_\varepsilon = \ker P_n$ for $\frac{1}{n} \leqslant \varepsilon$.

Now we consider the compactness condition used in [11] and [16] to define J_α :

2.7. $T \in J_\alpha$ *if and only if for every* $\varepsilon > 0$ *there is a set* $S \subset H$ *with cardinal power strictly less than* $\aleph_\alpha$ *such that for every* $x \in H$ *with* $\|x\| \leq 1$ *, there is* $s \in S$ *with* $\|Tx-s\| < \varepsilon$ *(in other words,* S *is an* ε*-net for* $\{Tx \; ; \; \|x\| \leq 1\}$*).*

Proof. Consider the case $\alpha > 1$. Let $T \in J_\alpha$ and $\{P_n\}$ as in 2.5. For given $\varepsilon > 0$, choose n large and set $S = \{TP_n x \; ; \; \|x\| \leq 1\}$. Now for $x \in H$ satisfying $\|x\| \leq 1$ if $s = TP_n x$ we have $\|Tx-s\| = \|Tx-TP_n x\| < \varepsilon$. Obviously the cardinal power of S is not larger than $\aleph_1$-rank $P_n < \aleph_\alpha$. If $\alpha = 1$, rank $P_n \leq \aleph_0$ and $S_1 = \{TP_n x; \|x\| \leq 1\}$ is separable, so choose for S a countable set dense in S_1. If $\alpha = 0$, we already observed that $J_\alpha \subset C(H)$, so that $\{Tx \; ; \; \|x\| \leq 1\}$ is relatively compact, and therefore totally bounded. In all cases, then, the "only if" part of 2.7 is proved. Consider the "if" part: assume T satisfies the condition in 2.7, and let K be the closed subspace generated by S, P the orthogonal projection on K. For $\|x\| \leq 1$ pick $s \in S$ with $\|Tx-s\| < \varepsilon$. Then $\|PTx-Tx\| \leq \|PTx-Ps\| + \|PS-Tx\| \leq \|P\| \; \|Tx-s\| + \|s-Tx\| < 2\varepsilon$, so PT tends to T. But the subspace K contains a dense subset of the same cardinal power as S (namely, the rational linear combinations of elements of S), and therefore rank P = dim K = card $S < \aleph_\alpha$, so that P , and PT, belong to J_α. Thus $T = \lim PT \in J_\alpha$, as desired.

§3. UNIVERSAL REPRESENTATIONS.

We recall (see [10] or [17, Ch. IV,V]) that if A is a C*-algebra with identity e and involution $x \to x^*$, A can be faithfully represented as a closed *-subalgebra of $B(H_A)$ for certain Hilbert space H_A. The description of H_A is as follows: Consider the set $L = \{p\}$ of all linear functionals $p: A \to C$, where C denotes the complex numbers, that are positive, i.e., $p(x^*x) \geq 0$ for all $x \in A$, and satisfy $p(e) = 1$. Such p will be called "states" of A. It can be seen that they are automatically continuous and that L is a convex subset of the dual of A as a Banach space. For $p \in L$, $a,b \in A$, define $(a,b) = p(b^*a)$; (a,b) is semi-bilinear (that is, linear in a and conjugate linear in b), $(a,a) \geq 0$ and also $|(a,b)|^2 \leq (a,a)(b,b)$ (Cauchy-Schwarz inequality). Factoring by the degeneracy set $N = \{a | (a,a) = 0\}$ we obtain an inner product space A/N whose completion is a Hilbert space to be denoted by H_p. Corresponding to each $a \in A$ there is an operator $a_p \in B(H_p)$

defined by extending $a_p(x+N) = ax + N$ by continuity from A/N to H_p. It is plain that $\|a_p\| \leq \|a\|$ and that $a \to a_p$ is a representation of A in H_p, i.e., a homomorphism of C*-algebras: $(ab)_p = a_p b_p$, $(\lambda a)_p = \lambda a_p$, $(a^*)_p = (a_p)^*$ for $a,b \in A$ and λ complex. The extreme points of L will be called "pure states". When p ranges on the set of pure states we obtain a family $\{H_p\}$ of Hilbert spaces and representations $a \to a_p \in B(H_p)$. However, different p_1, p_2 may determine equivalent representations $a \to a_{p_1}$, $a \to a_{p_2}$ in the sense that for some invertible $V: H_{p_1} \to H_{p_2}$ we have $Va_{p_1}V^{-1} = a_{p_2}$, and this is of course an equivalence relation. By selecting one p in each equivalence class we find a determining subset C of pure states. Then H_A is defined as $H_A = \sum_{p\in C}\oplus H_p$ and the representation $u\cdot A \to B(H_A)$ defined by $u(a) = \sum_{p\in C}\oplus a_p$ is called the *universal representation* of A. We aim to compute dim H_A when $A = B(H)/J$, for J a closed two-sided ideal of B(H). Observe that every two-sided ideal J of B(H) is a *-ideal in the sense that $T \in J$ implies $T^* \in J$ [5,Chap. 1, §1, N° 1] and therefore the quotients B(H)/J are C*-algebras when J is also closed.
In §2 we proved that the family $\mathcal{L}$ of closed two-sided ideals of B(H) is order isomorphic to the initial interval $[-1,\delta+1]$, where $\aleph_\delta = \dim H$. In particular J_δ is the largest proper two-sided ideal of B(H).

3.1. THEOREM. *Let* H *be a Hilbert space of dimension* $\aleph_\delta$ *and* J *a closed two sided ideal of* B(H) *different from the largest proper two sided ideal* J_δ. *Then the universal representation of* $A = B(H)/J$ *has dimension* $\aleph_{\delta+2}$.

The method of proof is suggested by a counting argument used in [15] (and credited to I. Kaplansky) that shows that $\dim H_A = 2^{2^{\aleph_0}}$ when A = B(H) and H is separable infinite dimensional.

We need some preliminaries.

Let X be a discrete topological space of cardinal power $\aleph_\nu$, and βX its Stone-Čech compactification. Then Card $\beta X = \aleph_{\nu+2}$ ([7, Ch. 3, Problem L, (c)], [21], [13]). Assume μ is an ordinal number and $\mu < \nu$. Denote by $X_\mu \subset \beta X$ the set $X_\mu = \cup\{\bar{S}\ ;\ S \subset X \text{ and Card } S \leq \aleph_\mu\}$, where $\bar{S}$ denotes the closure of S in βX.

3.2. LEMMA. *For every* $\mu < \nu$, Card $(\beta X - X_\mu) = \aleph_{\nu+2}$ *(where "-" denotes set theoretical difference).*

Proof. Choose $S \subset X$ with Card $S \leqslant \aleph_\mu$. It is easy to see that $\bar{S}$ is homeomorphic to βS (in fact, the injection i: $S \to \beta X$ extends to a mapping i': $\beta S \to \beta X$ and the image i'(βS) is compact and contains S as a dense subset),so that Card $\bar{S}=\aleph_{\mu+2}$.Thus Card $X_\mu \leqslant \aleph_{\mu+2}$.Card W, where W is the set of parts S of X with Card $S \leqslant \aleph_\mu$. Obviously Card $W \leqslant \aleph_\nu^{\aleph_\mu} \leqslant \aleph_{\nu+1}$, so that Card $X_\mu \leqslant \aleph_{\mu+2}\aleph_{\nu+1} = \aleph_{\nu+1}$, whence Card$(\beta X - X_\mu) = \aleph_{\nu+2}$ as desired.

Proof of Theorem 3.1. Let $J = J_\alpha$ with $\alpha < \delta$. Then $h(J) = \alpha < \alpha+1 = h(m_\alpha)$ and therefore $J \subset m_\alpha \neq B(H)$. Hence, if $B = B(H)/m_\alpha$, there is a natural homomorphism onto $A \to B$ (where $A = B(H)/J$). We shall show that there are $\aleph_{\delta+2}$ inequivalent pure states of B, a result that carries over to A by the homomorphism $A \to B$. This can be done as follows: let X be a discrete topological space with Card $X = \aleph_\delta$, and identify H with $\ell^2(X) = \{f: X \to C;\ \sum|f(x)|^2 < +\infty\}$. Clearly $\ell^\infty(X) = \{b: X \to C;\ \mathrm{Sup}|b(x)| < +\infty\}$ can be identified to a subalgebra D of B(H): the operator corresponding to b being $f(x) \to b(x)f(x)$. D is then the algebra of diagonal operators, and from a theorem of Krein [3, Ch. VI] or [17, Ch. V , §23 , N° 3, III], every pure state of D can be extended to a pure state of B(H). Clearly $\ell^\infty(X)$ can also be identified to $\mathcal{C}(\beta X)$, the Banach space of all complex valued continuous functions on the compact space βX, and each $x \in \beta X$ determines a positive functional p_x: $b \to \hat{b}(x)$, where $\hat{b} \in \mathcal{C}(\beta X)$ is the extension to βX of $b \in \ell^\infty(X)$. It can be seen that p_x is a pure state for every $x \in \beta X$ ([13], [24]) and in fact, these are all the pure states of $\ell^\infty(X)$. Denote again by p_x a pure state of B(H) extending p_x: $\ell^\infty(X) = D \to C$. Assume now that $x \in \beta X - X_\alpha$, and let $P \in D$ be the projection associated to the characteristic function b_S of some subset $S \subset X$ with Card $S = \aleph_\alpha$. It is clear from $x \notin \bar{S}$, that $\hat{b}_S(x) = 0$, or $p_x(P) = 0$, whence $p_x(P^*P) = p_x(P) = 0$ and therefore $P_{p_x} = 0$, which implies $T_{p_x} = 0$ for all $T \in m_\alpha$; this means that p_x induces a pure state of $B = B(H)/m_\alpha$, thus a pure state of $A = B(H)/J$. This shows that there are Card$(\beta X - X_\alpha) = \aleph_{\delta+2}$ different pure states of A. Clearly the members in an equivalence class of representations are in one-to-one correspondence with invertible operators $V \in B(H)$, which means that each class contains at most Card B(H) representations. Now from the matrix representation of operators follows that Card B(H) $\leqslant \aleph_{\delta+1}$, and therefore if $C = \{p\}$ contains a pure state in each equivalence class, we have Card $C.\aleph_{\delta+1} \geqslant$ Card$(\beta X - X_\alpha) = \aleph_{\delta+2}$, whence Card $C \geqslant \aleph_{\delta+2}$. Thus, since $H_A = \sum_{p \in C} \oplus H_p$, we have dim $H_A \geqslant$ Card $C \geqslant \aleph_{\delta+2}$. We need now estimates for dim H_A and Card L (L = set of all states). Clearly $L \subset C^{B(H)}$, so that Card $L \leqslant \aleph_1^{\delta+1} = \aleph_{\delta+2}$. Also, A/N, where $N = \{T ; p(T^*T) = 0\}$ is dense in H_p, so that dim $H_p \leqslant$ $\leqslant$ Card $A/N \leqslant$ Card $B(H) = \aleph_{\delta+1}$. Finally, dim $H_A \leqslant \sum_{p \in C} \dim H_p \leqslant$ $\leqslant$ Card $C . \aleph_{\delta+1} \leqslant$ Card $L . \aleph_{\delta+1} \leqslant \aleph_{\delta+2} \aleph_{\delta+1} = \aleph_{\delta+2}$. We conclude that dim $H_A = \aleph_{\delta+2}$, as claimed.

3.3. REMARK. This proof does not actually depend on the continuum hypothesis when $J = \{0\}$. Thus: "*if* dim $H = d$, *the dimension of the universal representation of* $B(H)$ *is equal to* 2^{2^d}". can be obtained without assuming that $2^{\aleph_\alpha} = \aleph_{\alpha+1}$.

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Recibido en julio de 1970.