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A SURVEY OF MODERN APPLICATIONS OF THE METHOD OF CONFORMAL MAPPING

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INTRODUCTION. Classical applications of conformal mapping to many stationary problems of mathematical physics go back over a century and continue to the present. These applications deal, in general, with solutions of Laplace's equation which remains invariant if the real plane is subjected to a conformal transformation. Consequently, any complicated configuration can be transformed into a more convenient one without modification of the governing partial differential equation.

Applications of conformal mapping techniques to the mathematical theory of elasticity are considerable more complex. The method is due to the great Russian mathematician Mushkelishvili. An excellent survey of applications of this method is available in the book *Elasticity and Plasticity* by J.N. Goodier and P.G. Hodge, Jr. (John Wiley and Sons, 1958).

It is the purpose of this paper to present a review of non-classical applications of conformal mapping in several fields of technology and applied sciences: acoustics, electromagnetic theory, vibrations, viscous flow problems, etc.

The present review should not be considered exhaustive but rather informative.

ELECTROMAGNETIC THEORY.

In general, exact analytical calculations of wavefields between conducting surfaces are only possible for configurations of simple shape, for example planes or cylinders having a circular cross-section. However, more complicated boundary configurations are needed in many technological applications.

It must be pointed out that the standard circular an rectangular waveguides do not satisfy all present and future requirements. Hence the need of investigating waveguides of very general cross section.

Since many curved surfaces can be transformed into simpler surfaces, the boundary configurations can be simplified using the method of conformal transformation.

When such a transformation is used the space in which the wave propagates becomes considerable more complex since the dielectric constant and the permeability become functions of position. In some cases the space becomes anisotropic ([1]).

Some authors feel that the complex transformed equations with their simplified boundary configurations are more suitable for numerical evaluations than the original differential system ([1]).

The most important contributions in this field are due to Meinke and his coworkers ([1],[2]). The curved boundary surfaces are transformed into parallel planes and rectangular coordinates are then used. The transformed governing differential system has the same structure for all transformed systems and they only differ in some position dependent factors which describe the non-uniformily of the field. The solution of the differential system is then expressed in terms of an infinite sum of orthogonal functions and a numerical evaluation of the equations is then possible.

The transformation expressions can be obtained by mathematical formulations, graphical methods or by utilizing the electrolytic tank technique. Other contributions in this field are due to Tischer ([3],[4]), Wohlleben ([5]), Chi and Laura ([6],[7]), Bava and Perona ([8]), Baier ([9]), etc.

It has been shown by Richter ([10]) that wave propagation around a cylinder represents an approximation to wave propagation around a sphere (e.g. the earth) when the influence of the curvature of the sphere perpendicular to the direction of propagation can be neglected. He proved that the conformal mapping applied to the cylinder yields an earth - flattening technique which agrees with first - order approximations obtained in spherical coordinates. It is interesting to point out that Pryce ([10]) treats wave propagation around the earth in spherical coordinates using a range transformation suggested by Pekeris and a height transformation suggested by Copson and that Richter proved that both independently proposed transformations follow directly from the application of conformal mapping ([10]).

Richter's paper is the only contribution which makes use of conformal mapping in a problem of radio propagation in the atmosphera.

FLOW AND HEAT TRANSFER IN DUCTS OF ARBITRARY SHAPE.

The analysis of the flow and heat transfer in ducts of arbitrary shape has been the subject of investigation for many years. A common application of such conduits is in space vehicles ([11]). A very general approach has been developed by Sparrow and Haji - Sheikh and several practical cases have been studied in an excellent paper ([11]).

A different approach has been followed in Refs. [12] and [13]. The technique introduced therein is not as general as that developed in Ref. [11] but it is more convenient for the specific problem tackled by the authors.

HEAT CONDUCTION PROBLEMS.

Laura and his coworkers have made use of conformal mapping techniques in the analysis of unsteady heat conduction problems in bars of arbitrary cross section ([14]-[16]).

Yu ([17]) has extended the method to deal with temperature dependent conductivity materials.

ANALYTICAL PREDICTION OF DRYING PERFOMANCE IN NON-CONVENTIONAL SHAPES. APLICATION TO THE DRYING OF APPLES.

The diffusional flow of water is an important part of many food drying processes. In general, several mechanisms are expected when considering a drying process. They are sometimes divided into two broad all-including cathegories: one in which drying occurs as if the system were pure water being evaporated, one with internal control. The first type may or may not occur but the second type is always present.

The internal control period of drying is described as a diffusional process, which follows Fick's second law in Reference [18]. Experimental data was obtained for a rectangular paralellepiped. This shape having natural coordinates, has a simple mathematical solution. Non-conventional shapes, which do not have natural coordinates on the x, y plane, are solved analitically by using conformal mapping. The initial and boundary conditions are those classical for a drying problem: uniform initial concentration, zero surface concentration. The transformed differential system is solved by the collocation along arcs procedure, obtaining moisture concentration distribution as a function of time and position.

For practical reasons, the total amount of moisture diffusing from the solid body is more useful than the concentration distribution. The solution is therefore integrated over the volume of the solid bodies.

The non-conventional cross sections considered are: cardicid, cylinder, corrugated, hexagon, epitrochoid, and square.

ION OPTICS.

The equation of trajectory of a charged particle in a two - dimensional electric field and a normal magnetic field has been derived by Naidu and Westphal ([19] and [20]) from the basic equation of motion. A series of paracial approximations reduced the non-lineal trajectory equation to a linear inhomogeneous ordinary differential equation.

The boundary value problem is solved by the Schwarz-Christoffel transformation and the field configuration is then found.

MATHEMATICAL THEORY OF ELASTICITY.

Only a few papers published after 1958 and which consequently have not been referenced in Goodier's excellent monograph will be briefly discussed in this section.

Florence and Goodier ([21]) have solved the thermoelastic problem of uniform heat flow disturbed by an insulated hole of ovaloid form. Deresiewicz ([22]) has extended the previous analysis to holes whose boundaries can be mapped conformally on a unit circle by means of polynomials.

The determination of stresses in beams subjected to pure bending and having holes of arbitrary shape has been analized in [23] and [24]. The cases of equilateral triangular, square, rectangular and regular polygonal holes have been examined in great detail and the circumferential normal stresses have been evaluated in each case as a function of the radius of curvature at the vertices ([23]).

The names of Wilson and Richardson must be, certainly, mentioned at this point since they have analyzed extremely complex shapes ([25],[26]). They have developed computer programs to find the corresponding mapping functions by solving an integral equation of the Fredholm type ([25]) or a system of coupled integral equations ([26]). A discussion on available methods for finding the mapping function has also been published ([48]).

A conformal mapping approach has also been used in predicting the stress distribution in rotating disks of non-circular shape ([27]). No numerical values are given.

Stress fields in plates with reinforced holes of several shapes have also been determined by several researchers ([28]-[30]).

References [31] and [32] deal with applications of conformal mapping to the analysis of stressed plates with inclusions.

Savin and his coworkers have followed a complex variable approach to determine stress fields around holes of arbitrary shape in physically nonlinear media ([33],[34]).

NON CLASSICAL APPLICATIONS OF CONFORMAL MAPPING TO FLUID FLOW PROBLEMS.

Bartels and Laporte ([35]) have developed a general method for treating the linearized equations of the supersonic flow past conical bodies. Their approach makes use of conformal mapping techniques.

Segel has shown that conformal mapping is a useful tool in obtaining the solution of certain unsteady two-dimensional perturbation problems involving the flow of a viscous incompressible fluid ([36]).

The case offlow between moving circular cylinders is solved by mapping the given eccentric - circular boundaries into concentric circles ([36]).

Yu and Chen ([37]) have solved the problem of an unsteady laminar flow of a viscous incompressible fluid in a conduit of arbitrary cross section due to a time dependent axial pressure gradient. The given region is transformed onto a unit circle to facilitate the choice of the coordinate functions. The stationary value problem in the circular region is then solved by the Rayleigh - Ritz method. Velocity profiles, friction factors and rates of energy dissipation factors are calculated for ducts of regular polygonal cross section.

SOLIDIFICATION PROBLEMS.

Siegel and coworkers ([38]-[40]) have developed a conformal mapping method for analyzing two-dimensional transient and steady - state solidification problems. The method has been applied to the solidification which takes place on a cold plate of finite width immersed in a flowing liquid and to the solidification inside of a cooled rectangular channel which containe a warm flowing liquid. The investigations have dealt with the transient and steady - state shapes of the frozen regions.

It is interesting to point out that the transient shapes of the frozen region are found by mapping the region into a potential plane and then determining the time varying conformal transformation between the potential and physical planes.

THEORY OF ACOUSTICS.

Acoustic waveguides of complicated cross section have been studied using conformal mapping techniques by several authors ([41]-[44]), ([46]-[48]). It must be pointed out that the governing differential system is similar to that of microwave theory:

$$\nabla^2 \phi + k^2 \phi = 0 \; ; \quad \phi = 0 \quad \text{(soft walls, TM waves)}$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{(rigid walls, TE waves)}$$

A method for solving three - dimensional axially symmetric problems related to the diffraction and radiation from a general class of bodies of revolution has recently been developed ([45]). The method depends on the conformal transformation of the region outside the meridian profile of the body onto the region outside a circle. The required boundary value problem is formulated in spherical coordinates in the transformal space. In this form, Galerkin's method can be applied to obtain a functional approximation for the solution of the boundary value problem.

THEORY OF PLATES.

Simply supported plates of rectilinear sides subjected to complex distributions of loading have been considered by Aggarwala in several papers ([49]-[51]). Aggarwala has derived a simple but very accurate formula which yields the central deflection of a simply supported centrally loaded rhombic plate. The formula, which depends only on the first coefficient obtained in the mapping of the rhombus onto a unit circle, gives results correct within about one per cent ([50]).

The approach is also valid to any simply supported plate of regular polygonal shape. Such functional relation is:

$$W_o \approx \frac{P}{8\pi D} \alpha_1^2$$

where W_o: deflection at the center, P: concentrated load, D:

flexural rigidity and α_1 : is the first coefficient of the mapping function given in series form:

$$z = \sum_{n=1}^{\infty} \alpha_n . \xi^n$$

Clamped and simply supported plates of complicated boundary shape have been considered by several authors ([52]-[54]).

In Ref.[55] Ramu determines the collapse load of plates of arbitrary boundary shape. The author uses a method similar to Musklelishvili for plane elasticity problems. By introducing a suitable stress function, the problem of determining a statically admissible stress field is reduced to finding a solution of the governing biharmonic equation in terms of analytic functions. It is assumed that the material obeys Von Mises yield condition.
As pointed out by Laura and Shahady ([56]) solution of the eigenvalue problem governing the stability of a thin elastic plate subjected to hydrostatic in-plate loading is easily accomplished when the boundary configuration is natural to one of the common coordinates systems. Reference [56] shows that is convenient to conformally transform the given domain onto a simpler one, i.e. the unit circle. The boundary conditions can then be satisfied identically. Since the governing partial differential equation is not invariant under the transformation and becomes considerably more complicated, a variational method is used to solve it. The method has been illustrated in the case of clamped and simply supported plates of various configurations.

It has also been shown that the determination of an upper bound of the critical in-plane loading of simply supported plates of rectilinear sides is quite straightforward if use is made of a theorem by Szego ([56]-[57]).

Obviously the same approach is also valid when determining natural frequencies of vibrations of plates of complicated boundary shape ([58]-[62]).

VIBRATIONS OF SOLID PROPELLANT ROCKET MOTORS.

The grain of a solid propellant rocket motor usually takes the form of a circular cylinder bonded to a thin case. This grain quite commonly has a star-shaped internal perforation. The mathematical solution of any boundary or eigenvalue problem becomes quite complicated in view of the exotic geometric configuration. This difficulty can be alleviated to a large extent by conformally transforming the grain cross section into a simpler region such as a circle or annulus. Several studies have been performed on axial shear vibrations of solid and hollow bars making use of conformal mapping and variational or bounding techniques ([63]-[65]).

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