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AN ATOMIC DECOMPOSITION OF DISTRIBUTIONS IN PARABOLIC H^p SPACES

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ABSTRACT . In this paper we extend the method employed by A.P.Calderón in [1] to obtain an atomic decomposition of distributions in parabolic H^p spaces with diagonalizable dilation groups, to a general parabolic H^p space. The main tool we use is a partition of unity of Whitney-type in this context.

1. PRELIMINARIES AND STATEMENT OF THE MAIN RESULT.

In this section we shall first state some background material of H^p spaces of A.P. Calderón and A. Torchinsky; we shall omit the corresponding proofs; they can be found in [1], [2] and [3].

We will be using the letter c to denote a constant which need not be the same in different ocurrences.

Let H^p, $0 < p \leqslant \infty$, be the classes of distributions in R^n introduced in [3]. That is: we consider a multiplicative group A_t of linear transformations of R^n such that if $x \in R^n$ and $|x|$ denotes its norm, then $t^\alpha |x| \leqslant |A_t x| \leqslant t^\beta |x|$, $1 \leqslant \alpha \leqslant \beta$, $t \geqslant 1$. The infinitesimal generator of A_t will be denoted with P. Let $d(x,y) = \rho(x-y)$ be the associated metric in R^n, where $\rho(x)$ is the unique value of t for which $|A_t^{-1}x| = 1$. Given φ in the class S of infinitely differentiable, rapidly decreasing functions in R^n and f in S', the class of tempered distributions, let

$$\varphi_t = t^{-\gamma} \varphi(A_t^{-1}x) \ , \quad \gamma = \text{trace } P \ ;$$

$$F(x,t) = (f * \varphi_t)(x) \ , \quad x \in R^n, \ t > 0$$

$$\text{and} \quad M_a(x,F) = \sup_{\rho(x-y) \leqslant at} |F(y,t)| \ , \ a > 0 \ .$$

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Then $f \in H^p$ if $M_a(x,F) \in L^p$, $0 < p \leqslant \infty$, $\hat{\varphi}(0) \neq 0$; and the norm of an element of H^p is defined by $\|f\|_{H^p} = \|M_a(x,F)\|_p$. As is well known the space H^p does not depend on the choice of φ or a, and for any two choices of φ and a the resulting norms are equivalent. For $0 < p \leqslant 1$ the distance function $\|f-g\|^p_{H^p}$ turns H^p into a complete metric space.

The transposed of A_t with respect to the ordinary inner product (x,y) in R^n will be denoted with A_t^*. The function ρ satisfies (see [2], 1.4)

(1.1) $\rho(A_t x) = t\,\rho(x)$.

(1.2) $\rho(x) \leqslant 1$ if and only if $|x| \leqslant 1$.

(1.3) $|x| \leqslant \rho(x)^\alpha$ if $|x|$ or $\rho(x) \leqslant 1$, $1 \leqslant \alpha$.

A k-atom α is a bounded function with compact support and with vanishing moments of all orders less than or equal to k. The p-norm of the atom α is defined as $\|\alpha\|_{(p)} = \|\alpha\|_\infty (\inf |B|^{1/p})$ where $|B|$ is the measure of a ball $B = \{x: \rho(x-x_0) \leqslant r\}$ containing the support of α. If $0 < p \leqslant 1$ and $k \geqslant (\gamma/p)-1$, then $\alpha \in H^p$ and $\|\alpha\|_{H^p} \leqslant c\,\|\alpha\|_{(p)}$, where the constant c depends on k and the choice of the norm in H^p.

We shall now state our result, which coincides in the diagonalizable case with the result obtained by A.P.Calderón in [1].

THEOREM. *Let* f *in* H^p, $0 < p \leqslant 1$ *and* $k \geqslant (\gamma/p)-1$. *Then there exists a sequence* α_j *of* k-*atoms, such that* $f = \sum_1^\infty \alpha_j$ *in* H^p, *that is* $\|f - \sum_1^N \alpha_j\|_{H^p} \longrightarrow 0$ *as* $N \longrightarrow \infty$, *and given a norm in* H^p *and the value of* k *there is a constant* c *such that*

$$c^{-1}\,\|f\|^p_{H^p} \leqslant \sum \|\alpha_j\|^p_{(p)} \leqslant c\,\|f\|^p_{H^p}\ .$$

2. We consider as in [1] an infinitely differentiable function ψ with compact support contained in $|x| \leqslant 1$ and with vanishing moments up to order k, with the property that $|\hat{\psi}(A_t^* x)|$ does not vanish identically in t for $x \neq 0$. Then there exists a function $\varphi(x) \in S$ such that $\hat{\varphi} \in C_0^\infty$ and $\hat{\varphi}(x) = 0$ near the origin and has the property

$$\int_0^\infty \hat{\varphi}(A_t^* x)\ \hat{\psi}(A_t^* x)\ \frac{dt}{t} = 1\ , \quad x \neq 0\ ; \tag{2.1}$$

(see [2], lemma 4.1).

Then the function

$$\hat{\eta}(x) = \int_1^\infty \hat{\varphi}(A_t^* x)\ \hat{\psi}(A_t^* x)\ \frac{dt}{t}\ , \quad x \neq 0$$

$$\hat{\eta}(0) = 1$$

is infinitely differentiable, has compact support, and equals 1 near the origin. Furthermore, as is readily verified

$$(2.2) \qquad \int_{t_0}^{t_1} \hat{\varphi}(A_t^* x)\ \hat{\psi}(A_t^* x)\ \frac{dt}{t} = \hat{\eta}(A_{t_0}^* x) - \hat{\eta}(A_{t_1}^* x)\ .$$

Given $f \in S'$ we define the functions

$$F(x,t) = (f * \varphi_t)(x)\ ,$$

and

$$G(x,t) = (f * \eta_t)(x)\ ,$$

where $\eta(x)$ is the inverse Fourier transform of $\hat{\eta}$.

If $f \in H^p$, then

$$M(x) = \sup_{\rho(x-y) \leqslant 3t} (|F(y,t)| + |G(y,t)|)$$

belongs to L^p.

Now, the distribution f can be expressed in terms of $F(x,t)$ as

$$f = \int_0^\infty \int F(y,t)\ \psi_t(x-y)\ dy\ \frac{dt}{t}\ ,$$

where the integral in t converges in the weak sense, that is

$$(2.3) \qquad \lim_{\substack{\varepsilon\to 0\\ \delta\to\infty}} \int \zeta(x) \int_\varepsilon^\delta F(y,t)\ \psi_t(x-y)\ dy\ \frac{dt}{t}\ dx = (f,\zeta)$$

for every ζ in S. This is seen by taking Fourier transforms. Furthermore, using (2.2) we have

$$(2.4) \qquad \int_{t_0}^{t_1} \int F(y,t)\ \psi_t(x-y)\ dy\ \frac{dt}{t} = G(x,t_0) - G(x,t_1)\ .$$

3. A PARTITION OF UNITY.

Let O be an open subset of R^n of finite measure. Then there exists a countable family of functions ξ_j with the following properties:

(1) $\forall\, x \in \mathcal{O}\ ,\ 0 \leqslant \xi_j(x)$ and $\sum_j \xi_j(x) = 1$.

(2) $\operatorname{supp}(\xi_j) \subset B_j$, where B_j is a ball of radius r_j i.e. $B_j = \{x\colon \rho(x-x_j) \leqslant r_j\}$, such that $d(B_j,\mathcal{O}^c) \leqslant c\, r_j$ and $\sum_j |B_j| \leqslant c\ |\mathcal{O}|$.

(3) $|\xi_j(x+z) - \xi_j(x)| \leqslant c\, \frac{1}{r_j}\, \rho(z)$, $\forall\ x,\ z \in R^n$.

Proof: For each integer k we consider the set Ω_k defined by

$$\Omega_k = \{x\colon\ 2^{-k-1} < d(x,\mathcal{O}^c) \leqslant 2^{-k}\}. \tag{3.1}$$

Since $|\mathcal{O}|$ is finite, we have $\mathcal{O} = \bigcup_{k\geq k_o} \Omega_k$. Let $\{B_{r'}(x)\}$ be a covering of Ω_k with balls $B_{r'}(x)$ of radius $r' = 2^{-k-5}$ and with centers in Ω_k. Then $E' = \bigcup_{k\geq k_o} \{B_{r'}(x)\}$ is a covering of $\mathcal{O}$ and the sup $\{\text{radius } (B_{r'}(x))\}$ is finite. Therefore, we can select a disjoint countable subfamily $\{B_{r'_j}(x_j)\}$ such that (see [2], lemma 1.6), $\mathcal{O} \subset \bigcup_j B_{3r'_j}(x_j)$.

We take $r_j = 4\, r'_j$ and $B_j = B_{r_j}(x_j)$, that is, the ball with center x_j and radius r_j. Then $E = \{B_j\}$ satisfies

(i) $\mathcal{O} = \bigcup_j B_j$

(ii) $\sum_j |B_j| \leqslant c\ |\mathcal{O}|$

(iii) $d(B_j,\mathcal{O}^c) \leqslant c\ r_j$

(iv) Let $B_j \in E$. Then there are at most N balls in E which intersect B_j. N depends only on the dimension.

Let $B_{r_j}(x_j)$ be a ball of E, then there exists k such that $x_j \in \Omega_k$ and $r_j = 2^{-k-3}$, therefore if $z \in B_{r_j}(x_j)$ we have

$$2^{-k-1} - 2^{-k-3} \leqslant d(z,\mathcal{O}^c) < 2^{-k} + 2^{-k-3}\ . \tag{3.3}$$

Now, from (3.3) we have $B_{r_j}(x_j) \subset \mathcal{O}$, and using (3.2) we obtain (i). Observe that $\sum |B_{r_j}(x_j)| = 4^\gamma \sum |B_{r'_j}(x_j)| \leqslant 4^\gamma\ |\mathcal{O}|$, which is (ii).

Also (iii) follows from (3.3), with $c = 2^3+1$.

In order to prove (iv), we note that, on account of (3.3) if $B_{r_{j'}}(x_{j'})$ is a ball which intersects $B_{r_j}(x_j)$, then $r_{j'}$ must be one of the values $2r_j$, r_j or $2^{-1}r_j$ and therefore the corresponding $B_{r'_{j'}}(x_{j'})$, $(r'_{j'} = 4^{-1}r_{j'})$, is contained in the ball $B_{4r_j}(x_j)$. Since the balls $B_{r'_j}(x_j)$ are disjoint, we must have

$$N\,\omega_n\,(8^{-1}r_j)^\gamma \leqslant \omega_n\,(4r_j)^\gamma$$

where N is the number of balls which intersect B_j, ω_n the measure of the unit ball. The desired result follows from this inequality.

Let now, $\xi(x)$ be a function in D with the properties: $0 \leqslant \xi \leqslant 1$, $\xi(x) = 1$ for $\rho(x) \leqslant 1$ and $\xi(x) = 0$ for $\rho(x) \geqslant 1 + 3^{-1}$. We define

$$\xi'_j(x) = \xi(A^{-1}_{3r'_j}(x-x_j))$$

where x_j is the center of B_{r_j}, $r_j = 4r'_j$;

$$\Xi(x) = \sum \xi'_j(x) \qquad \text{and} \qquad \xi_j(x) = \frac{\xi'_j(x)}{\Xi(x)} .$$

Notice that $\xi'_j(x) = 1$ for x in $B_{3r'_j}(x_j)$ and $\xi'_j(x) = 0$ for x in the complement of $B_{r_j}(x_j)$, and, according to (iv) we have $1 \leqslant \Xi(x) \leqslant N$ thus the functions ξ_j are well defined, they belong to D, $0 \leqslant \xi_j$, $\sum\xi_j(x) = 1$ for x in $\mathcal{O}$, and $\mathrm{supp}(\xi_j) \subset B_j$. Now these properties combined with (i),(ii) and (iii) give (1) and (2).

Finally we shall prove (3). Let $\rho(A^{-1}_{3r'_j}x) < 1$. We observe that

$$\xi_j(x) = \frac{\xi(A^{-1}_{3r'_j}(x-x_j))}{\sum_i \xi(A^{-1}_{3r'_i}(x-x_i))} ,$$

where i takes the values for which B_i intersects B_j, and x_i is the center of B_i. As we have shown there are at most N values of i, and r'_i must be r'_j or $2r'_j$ or $2^{-1}r'_j$. On the other hand, the function

$$\frac{\xi(y - A^{-1}_{3r'_j}x_j)}{\sum_i \xi(A^{-1}_{a_i}(y - A^{-1}_{3r'_j}x_i))} ,$$

where i takes the same values as before, and $a_i = 1, 2$ or 2^{-1} for $r_i' = r_j'$, $2r_j'$ or $2^{-1}r_j'$; has bounded derivatives, and the bound c depends only on N, ξ, and the group A_t. Therefore

(3.4) $|\xi_j(x+z) - \xi_j(x)| =$

$$= \left| \frac{\xi(A^{-1}_{3r_j'}x + A^{-1}_{3r_j'}z - A^{-1}_{3r_j'}x_j)}{\sum_i \xi(A^{-1}_{a_i}(A^{-1}_{3r_j'}x + A^{-1}_{3r_j'}z - A^{-1}_{3r_j'}x_i))} - \frac{\xi(A^{-1}_{3r_j'}x - A^{-1}_{3r_j'}x_j)}{\sum_i \xi(A^{-1}_{a_i}(A^{-1}_{3r_j'}x - A^{-1}_{3r_j'}x_i))} \right| \leqslant$$

$$\leqslant c\, |A^{-1}_{3r_j'}z| .$$

Since $\rho(A^{-1}_{3r_j'}z) < 1$, then $|A^{-1}_{3r_j'}z| < 1$, and using (1.3) we have

$$|A^{-1}_{3r_j'}z| \leqslant [\rho(A^{-1}_{3r_j'}z)]^{\alpha} \leqslant \rho(A^{-1}_{3r_j'}z). \tag{3.5}$$

In the other case, that is $\rho(A^{-1}_{3r_j'}z) \geqslant 1$, since $\xi_j \leqslant 1$, we have

$$|\xi_j(x+z) - \xi_j(x)| \leqslant 2\rho(A^{-1}_{3r_j'}z). \tag{3.6}$$

Now, using the property (1.1) of ρ, from (3.4), (3.5) and (3.6) it follows that

$$|\xi_j(x+z) - \xi_j(x)| \leqslant c\rho(A^{-1}_{3r_j'}z) \leqslant c\,\frac{1}{r_j}\,\rho(z)$$

as we wished to show.

4. The function $M(x)$ is lower semicontinuous and consequently the sets

$$\mathcal{O}_i = \{x : M(x) > 2^i\}$$

are open.

Let ξ_{ij} be the functions of the partition of unity in $\mathcal{O}_i$. We consider also the subsets of $R^n \times R^+$

$$\tilde{\mathcal{O}}_i = \{(x,t) : 0 < 2t < d(x, \mathcal{O}_i^c)\}$$

where $d(x, \mathcal{O}_i^c)$ denotes the distance between x and the complement

O_i^c of O_i, and let $\chi_i(x,t)$ be the characteristic function of the set $\tilde{O}_i - \tilde{O}_{i+1}^c$.

Now we define the functions

$$\alpha_{ij}^{(\varepsilon)}(x) = \int_{\varepsilon}^{\infty} \int \xi_{ij}(y)\ \chi_i(y,t)\ F(y,t)\ \psi_t(x-y)\ dy\ \frac{dt}{t} \quad ,$$

$$\alpha_{ij}(x) = \lim_{\varepsilon\to 0} \alpha_{ij}^{(\varepsilon)}(x) \quad , \quad (p,p) \; .$$

We shall prove that the $\alpha_{ij}(x)$ are k-atoms which give the desired decomposition of f.

(a). We shall first estimate the supports of the $\alpha_{ij}^{(\varepsilon)}$. Let B_{ij} be the balls considered in 3.(2) such that $\operatorname{supp}(\xi_{ij}) \subset B_{ij}$ and r_{ij} the radius of B_{ij}. Then if $y \in B_{ij}$ we have $d(y,O_i^c) \leqslant c\ r_{ij}$; therefore $\chi_i(y,t) = 0$ if $t \geqslant \frac{c}{2}\ r_{ij}$, for all $y \in B_{ij}$. Since $\operatorname{supp}(\psi_t(x-y)) \subset \{x:\rho(x-y) \leqslant t\}$, if $B_{ij} \cap \{\rho(x-y) \leqslant t\} \neq \emptyset$ and $\chi_j(y,t) \neq 0$, we must have $d(x,B_{ij}) \leqslant t \leqslant \frac{c}{2}\ r_{ij}$. Consequently

$$\operatorname{supp}(\ \alpha_{ij}^{(\varepsilon)}\ ) \subset B_{c'r_{ij}} \quad ,$$

where $B_{c'r_{ij}}$ is the ball with the same center, and radius $c'r_{ij}$.

(b). We shall now estimate the sizes of $\alpha_{ij}^{(\varepsilon)}$. Given $x \in R^n$, consider the following values of t:

$$t_o = \frac{d(x,O_{i+1}^c)}{3} \; , \; \bar{t}_o = d(x,O_{i+1}^c) \; , \; t_1 = \frac{d(x,O_i^c)}{3} \; , \; \bar{t}_1 = d(x,O_i^c),$$

then $0 \leqslant t_k \leqslant \bar{t}_k$, $\bar{t}_k \leqslant 3t_k$, $k = 0$, 1 ; and

$$\text{(4.2)} \quad \begin{aligned} &\chi_i(y,t)\psi_t(x-y) = 0 \quad \text{for} \quad t < t_o,\ t_o \neq 0 \quad \text{or} \quad 0 \leqslant \bar{t}_1 < t, \\ &\chi_i(y,t)\psi_t(x-y) = \psi_t(x-y) \quad \text{for} \quad 0 \leqslant \bar{t}_o \leqslant t < t_1,\ t_1 \neq 0,\ t \neq 0. \end{aligned}$$

Observe that $\chi_i(y,t) = \chi^{(1)}(y,t)\ \chi^{(2)}(y,t)$, where $\chi^{(1)}(y,t)$ is the characteristic function of $\tilde{O}_i$ and $\chi^{(2)}(y,t)$ is the characteristic function of $\tilde{O}_{i+1}^c$. Now if $y \in \operatorname{supp}(\psi_t(x-y))$ then $\rho(x-y) \leqslant t$, and, we have

$$d(y,O_i^c) \leqslant d(x,O_i^c) + t \leqslant 2t \; , \quad \text{for} \quad 0 \leqslant \bar{t}_1 < t,$$

$$d(y,O_i^c) \geqslant d(x,O_i^c) - t > 2t \; , \quad \text{for} \quad t < t_1,\ t_1 \neq 0 \; ;$$

and consequently

$$\chi^{(1)}(y,t)\ \psi_t(x-y) = 0 \quad \text{for} \quad 0 \leqslant \overline{t}_1 < t$$

$$\chi^{(1)}(y,t)\ \psi_t(x-y) = \psi_t(x-y) \quad \text{for} \quad t < t_1,\ t_1 \neq 0.$$

Similarly, we find that

$$\chi^{(2)}(y,t)\ \psi_t(x-y) = 0 \quad \text{for} \quad t < t_o,\ t_o \neq 0$$

$$\chi^{(2)}(y,t)\ \psi_t(x-y) = \psi_t(x-y) \quad \text{for} \quad 0 \leqslant \overline{t}_o \leqslant t\ ,\ t \neq 0.$$

From these estimates we obtain (4.2).

First, we will estimate the integral (4.1) over the intervals $(a,\overline{t}_k)$, $t_k \leqslant a < \overline{t}_k$, $k = 0,1$. We notice that if $\chi_i(y,t) \neq 0$ then $(y,t) \in \widetilde{O}^c_{i+1}$ and therefore $|F(y,t)| \leqslant 2^{i+1}$. Also $\int |\psi_t(x-y)|\ dy$ is a constant independent of t and x, and because $\overline{t}_k \leqslant 3t_k$, we have

$$(4.3) \qquad \left| \int_a^{\overline{t}_k} \int \xi_{ij}(y)\ \chi_i(y,t)\ F(y,t)\ \psi_t(x-y)\ dy\ \frac{dt}{t} \right| \leqslant$$

$$\leqslant 2^{i+1} \int_{t_k}^{\overline{t}_k} \left[\int |\psi_t(x-y)|\ dy\right] \frac{dt}{t} \leqslant c\ 2^i.$$

Now , we shall estimate the integral (4.1) over the interval (a,t_1) for $0 < \overline{t}_o \leqslant a < t_1$ or $0 = t_o < a < t_1$. We have

$$(4.4) \qquad \int_a^{t_1} \int \xi_{ij}(y)\ F(y,t)\ \psi_t(x-y)\ dy\ \frac{dt}{t} =$$

$$= \int_a^{t_1} \int [\xi_{ij}(y) - \xi_{ij}(x)]\ F(y,t)\psi_t(x-y)\ dy\ \frac{dt}{t}\ +$$

$$+\ \xi_{ij}(x) \int_a^{t_1} \int F(y,t)\ \psi_t(x-y)\ dy\ \frac{dt}{t}\ .$$

Using (2.4) we find that the last term is equal to

$$(4.5) \qquad \xi_{ij}(x)\ [G(x,t_1) - G(x,a)]$$

and since (x,t_1) and $(x,a) \notin \widetilde{O}_{i+1}$, we have $|G(x,t_1)| \leqslant 2^{i+1}$, $|G(x,a)| \leqslant 2^{i+1}$, and consequently the absolute value of this term is less than or equal to $c2^i$. Setting $z = x-y$ in the first term,we obtain

$$(4.6)\qquad \int_a^{t_1}\int [\xi_{ij}(x+z) - \xi_{ij}(x)]\ F(x+z,t)\ \psi_t(z)\ dz\ \frac{dt}{t}\ ;$$

now $|F(x+z,t)| \leqslant 2^{i+1}$ and on account of the property (3) of the ξ_{ij}, (4.6) is dominated in absolute value by

$$(4.7)\qquad c\ 2^{i+1}\ \frac{1}{r_{ij}}\int_a^{t_1}\int \rho(z)\ |\psi_t(z)|\ dz\ \frac{dt}{t}\ .$$

We saw in (a) that $\operatorname{supp}(\alpha_{ij}^{(\varepsilon)}(x)) \subset B_{cr_{ij}}$ which implies that $t_1 \leqslant c\ r_{ij}$; on the other hand $\frac{1}{t}\rho(z) = \rho(A_t^{-1}z)$ then $\int \frac{\rho(z)}{t}\ |\psi_t(z)|dz$ is a constant independent of t and x. Therefore we have that (4.7) is less than or equal to

$$(4.8)\qquad c\ 2^{i+1}\ \frac{1}{r_{ij}}\int_0^{cr_{ij}} dt \leqslant c\ 2^i\ .$$

Which combined with the previous estimate give $|\alpha_{ij}^{(\varepsilon)}(x)| \leqslant c\ 2^i$.

(c). The fact that the moments up to order k of $\alpha_{ij}^{(\varepsilon)}$ vanish, follows by integration from the vanishing of the moments of $\psi_t(x-y)$ as a function of x.

(d). We shall show now, that the same results (a), (b) and (c), are valid for α_{ij}. In order to obtain this, it suffices to prove that $\alpha_{ij}^{(\varepsilon)}$ converges almost everywhere. In fact, if $\bar{t}_1 = 0$ or $t_o \neq 0$, then $\alpha_{ij}^{(\varepsilon)}(x)$ is constant, otherwise, since (4.5) converges almost everywhere as $a \to 0$ and the integral (4.6) converges absolutely in $(0,t_1)$, we find that (4.4) converges as $a \to 0$ almost everywhere and the desired result follows.

From the results above we have

$$(4.9)\qquad \|\alpha_{ij}\|^p_{(p)} \leqslant c\ 2^{ip}\ |B_{ij}|\ ,$$

and we obtain

$$(4.10)\qquad \sum\|\alpha_{ij}\|^p_{(p)} \leqslant c\sum 2^{ip}|B_{ij}| \leqslant c\sum 2^{ip}\ |O_i| \leqslant$$

$$\leqslant (1-2^{-p})^{-1}\sum 2^{ip}(|O_i| - |O_{i+1}|) \leqslant (1-2^{-p})^{-1}\|M(x)\|_p^p \leqslant c\ \|f\|^p_{H^p}$$

Thus, since $\|\alpha_{ij}\|^p_{H^p} \leqslant c\ \|\alpha_{ij}\|^p_{(p)}$, the series $\sum\alpha_{ij}$ converges in the metric of H^p ; denoting by g its sum, we have

$$\|g\|_{H^p}^p \leqslant \sum \|\alpha_{ij}\|_{(p)}^p \leqslant c\,\|f\|_{H^p}^p .$$

Finally we shall show that $g = f$. Observe that the functions $\alpha_{ij}^{(\varepsilon)}$ satisfy (4.9), and then (4.10) for each ε i.e. $\sum \|\alpha_{ij}^{(\varepsilon)}\|_{(p)}^p \leqslant c\,\|f\|_{H^p}^p$. Let $g^{(\varepsilon)} = \sum \alpha_{ij}^{(\varepsilon)}$ in H^p. On the other hand, from the properties (a),(b),(c) and the convergence almost everywhere of $\alpha_{ij}^{(\varepsilon)}$ to α_{ij}, it follows that $\|\alpha_{ij}^{(\varepsilon)} - \alpha_{ij}\|_{H^p} \longrightarrow 0$ as $\varepsilon \longrightarrow 0$. Then

$\|g^{(\varepsilon)} - g\|_{H^p} \longrightarrow 0$ as $\varepsilon \longrightarrow 0$.

Now, if $\zeta(x) \in S$, we have

$$(g^{(\varepsilon)},\zeta) = \sum(\alpha_{ij}^{(\varepsilon)},\zeta) = \sum \int \zeta(x)\int_\varepsilon^\infty \int \xi_{ij}(y)\chi_i(y,t)F(y,t)\psi_t(x-y)\,dy\,dt$$

Since $|F(y,t)| \leqslant c\, t^{-\gamma/p}$ and $\sum\xi_{ij}(y)\chi_i(y,t) = 1$, we may sum under the integral sign, and using (2.3) we obtain

$$(g,\zeta) = \lim_{\varepsilon\to 0}\ (g^{(\varepsilon)},\zeta) = (f,\zeta) ,$$

which implies $g = f$. This concludes the proof of the theorem.

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