REDOXION OF CODIMENSION OF ISOMETRIC IMMERSIONS BETWEEN INDEFINITE RIEMANNIAN MANIFOLDS

Marcos Dajczer

1. INTRODUCTION.

Let \( f: M^n_s \rightarrow Q^{n+p}(c) \) be an isometric immersion of a connected indefinite Riemannian manifold of dimension \( n \) and signature \( (s,n-s) \) into an indefinite manifold of constant curvature \( c \). If \( s=1 \) or \( s=n-1 \), we say that \( M^n_s \) is a Lorentz manifold. By changing the sign in the inner products we may assume that \( s=1 \). We say that the immersion \( f \) is \( m \)-regular if the \( k \)th normal space of the immersion \( N_k \) satisfies: \( \dim N_k = \) constant for \( k = 1, \ldots, m \) (see Section 2 for further definitions). The aim of this paper is to extend the main result of [1] to the indefinite Riemannian case. We prove the following result

1.1. THEOREM. Let \( f: M^n_s \rightarrow Q^{n+p}(c) \) be an isometric immersion.
Assume that the curvature tensor of the normal connection satisfies
\[
(V^1)^m N^1 | N_m = 0
\]
and that the mean curvature vector satisfies
\[
(V^1)^m H \subset N_m.
\]
Then there exists a totally geodesic submanifold \( Q^k \) of \( Q^{n+p}(c) \) of dimension \( n+k \), where \( k = \dim N_m \), such that \( f(M^n) \subset Q^k \).

2. PRELIMINARIES.

We denote by \( M^n_s \) a differentiable manifold whose tangent spaces have a nondegenerate metric of signature \( (s,n-s) \). Let us consider an isometric immersion, \( f: M^n_s \rightarrow \tilde{M}^{n+p} \), of one indefinite Riemannian manifold into another. Given \( p \in M \), we identify the tangent space \( T_p M \) to \( M \) at \( p \) with \( \tilde{df}(T_p M) \). The normal space \( T_p \tilde{M} \) is the subspace of \( T_p \tilde{M} \) consisting of all vectors \( \xi(p) \in T_p \tilde{M} \) which are normal to \( T_p M \) with respect to the metric \( \langle , \rangle \) of \( \tilde{M} \). Let \( \overline{V} \) (resp. \( V \)) be the co-
variant differentiation of the Levi-Civita connection in \( \tilde{M} \) (resp. \( M \)) and \( \nabla^\perp \) the covariant differentiation in the normal bundle of \( f \).

Given \( \xi(p) \in T^*_p M \), we define the second fundamental form of \( f \) relative to \( \xi(p) \)

\[
A_\xi(p): T_p \tilde{M} \to T_p M
\]

by the Weingarten equation:

\[
\nabla_X \xi = -A_\xi X + \nabla^\perp_X \xi,
\]

where \( X \in T_p M \) and \( \xi \) is any normal extension of \( \xi(p) \).

We shall denote the curvature tensor of \( \nabla \) by \( R \) and that of \( \nabla^\perp \) by \( R^\perp \), i.e.

\[
R(X,Y) = \nabla_X \nabla_Y - \nabla_Y \nabla_X - [X,Y]
\]

and

\[
R^\perp(X,Y) = \nabla^\perp_X \nabla^\perp_Y - \nabla^\perp_Y \nabla^\perp_X - \nabla^\perp[X,Y]
\]

We define the bilinear symmetric form

\[
\alpha: T_p M \times T_p M \to T_p M^\perp
\]

by the Gauss equation:

\[
\nabla_X Y = \nabla_X Y + \alpha(X,Y)
\]

Then, the condition

\[
<\alpha(X,Y), \xi> = <A_\xi X,Y>
\]

is satisfied.

If the ambient space has constant curvature, the following relations hold:

\[
(\nabla_X A)_\xi(Y) = (\nabla_Y A)_\xi(X), \quad \text{Codazzi's equation}
\]

and

\[
<R^\perp(X,Y) , \xi> = <[A_\xi, A_\eta] X, Y>, \quad \text{Ricci's equation}
\]

A basis \( X_1, \ldots, X_n \) of an indefinite inner product space with signature \((s, n-s)\) is called orthonormal if

\[
<X_i, X_j> = -\delta_{ij}, \quad 1 \leq i, j \leq s,
\]

\[
<X_r, X_t> = \delta_{rt}, \quad s+1 \leq r, t \leq n \text{ and } \quad <X_1, X_r> = 0, \quad 1 \leq i \leq s, \quad s+1 \leq r \leq n.
\]

If the vector space is a Lorentz space, then a pseudo-orthonormal basis is one of the form \( Z, \tilde{Z}, X_1, \ldots, X_{n-2} \), such that

\[
<Z, Z> = 0 = <\tilde{Z}, \tilde{Z}>, \quad <Z, \tilde{Z}> = 1, \quad <X_i, Y_j> = \delta_{ij}, \quad 1 \leq i, j \leq n-2 \text{ and } <Z, X_1> = 0 = <\tilde{Z}, X_1> \quad 1 \leq i \leq n-2.
\]

We define the mean curvature vector as
where $X_1, \ldots, X_n$ is an orthonormal basis of $T_pM$.

We say that the immersion is **totally geodesic** if $\alpha_\xi = 0$ for all $\xi \in T^*M$.

Given $p \in M$, we define the *first normal space* as

$$N_1(p) = \text{Span} \{a(X,Y)(p) : X,Y \in T_pM\}.$$  

We define the $k^{th}$ normal space as

$$N_k(p) = \text{Span} \{a(X,Y)(p), \nabla_{w_1} a(X,Y)(p), \ldots, \nabla_{w_{k-1}} \nabla_{w_1} a(X,Y)(p)\}$$

for $k = 2, 3, \ldots$, where $X,Y,w_1,\ldots,w_{k-1}$ are vector fields tangent to $M$.

A normal subbundle of dimension $k$ is a family $L(p)$, for all $p \in M$, of vector subspaces of $T_p^*M$ of dimension $k$ with the property that, for all $q \in M^n$, there are an open neighborhood $U$ of $q$ and $k$ differentiable fields $\xi_1, \ldots, \xi_k$ defined in $U$ such that, for all $p \in U$, $\xi_1(p), \ldots, \xi_k(p)$ generate $L(p)$.

An immersion is said to be *m-regular* if each $N_k(p)$, for $k = 1, \ldots, m$ and for all $p \in M$, has constant dimension. It is easily seen that if an immersion is $m$-regular, then each $N_k$ for $k = 1, \ldots, m$ is a normal subbundle.

If $L$ is a normal subbundle, by $(\nabla\xi)^m R^L = 0$ it is to be understood that

$$((\nabla\xi)^m R^L)(X_1;X_2;\ldots;X_{m+2})(\xi) = 0$$

for all $X_1, \ldots, X_{m+2} \in TM$ and all $\xi \in L$.

Finally, if $\eta$ is a section of the normal subbundle $L$, then $(\nabla\xi)^m \eta \in L$ means that

$$\nabla_{X_1} \ldots \nabla_{X_m} \eta \in L \text{ for all } X_1, \ldots, X_m \in TM.$$  

3. PROOF OF THEOREM 1-1.

First, we recall the following indefinite version of a theorem of Allendoerfer-Erbacher (see [2], [3]).

3-1 PROPOSITION. Let $f : M^n \rightarrow Q^{n+p}_{c}(c)$ be an isometric immersion of a connected indefinite Riemannian manifold into a space form. If the
There exists a k-dimensional parallel normal subbundle \( L(p) \) which contains the first normal space \( N_1(p) \) for all \( p \in M^n \), then there exists a \((n+k)\)-dimensional totally geodesic submanifold (possible degenerate) \( \mathcal{Q}^n \) of \( \mathcal{Q}^{n+k}(c) \) such that \( f(M^n) \subset \mathcal{Q}^n \).

The following result is the main part of the proof of Theorem 1.1.

3-2. PROPOSITION. Let \( f: M^n \rightarrow Q^{n+k}(c) \) be an isometric immersion that is \( m \)-regular in an open neighborhood \( U \) of a point \( p \) of \( M^n \). Then

\[
N_{m+1}^1(p) = \{ \xi \in N_1^1(p): ((\nabla^1)^k R^1(\xi)) = 0, (\nabla^1)^k H(p) \perp \xi \text{ for } 0 \leq k \leq m \}.
\]

The proof of the following four lemmas is the same as in the positive definite case (see [1]).

3-3. LEMMA. Let \( M \) be an indefinite Riemannian manifold and \( \xi, \eta \) vector fields defined in an open neighborhood \( U \) of a point \( p \) of \( M \). Then, we have that

i) \( <(\nabla^1)^k \xi, \eta> = 0 \text{ for } 0 \leq k \leq m \) if and only if \( <\xi, (\nabla^1)^k \eta> = 0 \text{ for } 0 \leq k \leq m \).

ii) \[
\begin{cases}
<\xi, (\nabla^1)^k \eta> = 0 \text{ for } 0 \leq k \leq m \\
<\xi, (\nabla^1)^{m+1} \eta>(p) = 0.
\end{cases}
\]

if and only if

\[
\begin{cases}
<\xi, (\nabla^1)^k \eta> = 0 \text{ for } 0 \leq k \leq m \\
<\xi, (\nabla^1)^{m+1} \eta>(p) = 0.
\end{cases}
\]

3-4. LEMMA. Let \( f: M \rightarrow \tilde{M} \) be an isometric immersion that is \( m \)-regular in an open set \( U \) of \( M \). Then, in \( U \), we have that for \( r = 1, \ldots, m \)

\( \xi \in N_r^1 \) if and only if \( A_{(\nabla^1)^k} \xi = 0 \text{ for } 0 \leq k \leq r-1 \).

3-5. LEMMA. Let \( f: M \rightarrow \tilde{M} \) be an isometric immersion that is \( m \)-regular in an open neighborhood \( U \) of a point \( p \) of \( M \). Then,

\( \eta_p \in N_{m+1}^1(p) \) if and only if there exists a local extension \( \eta \) of \( \eta_p \) such that i) \( A_{(\nabla^1)^k} \eta = 0 \text{ for } 0 \leq k \leq m-1 \) and ii) \( A_{(\nabla^1)^m} \eta(p) = 0 \).

3-6. LEMMA. Let \( f: M \rightarrow \tilde{M} \) be an isometric immersion and \( \eta \) a normal vector field defined in an open neighborhood \( U \) of a point \( p \) of \( M \). Then, we have that

i) \( ((\nabla^1)^k R^1)(\eta) = 0 \text{ for } 0 \leq k \leq m-1 \) if and only if
$R^l((\gamma^l)^k \eta) = 0$ for $0 \leq k \leq m-1$.

ii) \[
\begin{cases}
((\gamma^l)^k R^l)(\eta) = 0 & 0 \leq k \leq m-1 \\
((\gamma^l)^m R^l)(\eta(p)) = 0.
\end{cases}
\]

if and only if
\[
\begin{cases}
R^l((\gamma^l)^k \eta) = 0 & 0 \leq k \leq m-1 \\
R^l((\gamma^l)^m \eta(p)) = 0.
\end{cases}
\]

PROOF OF PROPOSITION 3-2. First of all, we note that if $\xi \in N_1^l(p)$, then $R^l(\xi) = 0$ and $H(p) \perp \xi$. The proof will be divided in two parts, each of them showing one of the inclusions.

i) Let $\eta_p \in N_1^{m+1}(p)$. By Lemma 3-5, there exists a local extension $\eta$ of $\eta_p$ such that

$A_{(\gamma^l)^k} \eta = 0$ for $0 \leq k \leq m-1$ and $A_{(\gamma^l)^m} \eta(p) = 0$.

By the Ricci equation, we may write

$R^l((\gamma^l)^k \eta) = 0$ for $0 \leq k \leq m-1$ and $R^l((\gamma^l)^m \eta(p)) = 0$.

By Lemma 3-6 it follows that

3-7 $((\gamma^l)^k R^l)(\eta_p) = 0$ for $0 \leq k \leq m$.

By definition, one has that $H \in N_1$. Therefore, it is immediate that

3-8 $(\gamma^l)^k H(p) \perp \eta_p$ for $0 \leq k \leq m$.

Then, the first inclusion follows from (3-7) and (3-8).

ii) For this part of the proof we shall use induction. Using i), we may suppose that the proposition holds for $N_j^l(p)$ for $1 \leq j \leq h$. Let $\eta_p \in N_j^l(p)$, which satisfies

3-9 $((\gamma^l)^k R^l)(\eta_p) = 0$ for $0 \leq k \leq h$

and

3-10 $(\gamma^l)^k H(p) \perp \eta_p$ for $0 \leq k \leq h$.

By the induction hypothesis together with (3-9) and (3-10), we thus have $\eta_p \in N_h^l(p)$. Let $\eta$ be a local extension of $\eta_p$ in $N_h^l$. Then,

3-11 $((\gamma^l)^k R^l)(\eta) = 0$ for $0 \leq k \leq h-1$

and

3-12 $(\gamma^l)^k H \perp \eta$ for $0 \leq k \leq h-1$.

From Lemma 3-6, (3-9) and (3-11), it follows that
But this means that
\[ <K(X,Y)(\nu^\perp_{z_h} \ldots \nu^\perp_{z_1} \eta(p)), \xi> = 0 \] for all \( X,Y,z_1,\ldots,z_h \in TM \),
\[ \xi \in T_pM^1. \]

By Ricci's equation
\[ <[A_{\nu^\perp_{z_h}} \ldots A_{\nu^\perp_{z_1}} \eta(p)], A_{\nu^\perp_{z_h}} X, Y> = 0. \]

In particular,
\[ 3-13 \quad [A_{\nu^\perp_{z_h}} \ldots A_{\nu^\perp_{z_1}} \eta(p), A_{\nu^\perp_{z_h}} \ldots A_{\nu^\perp_{z_1}} \eta(p)] = 0 \] for all
\[ z_1,\ldots,z_h, y_1,\ldots,y_h \in TM. \]

On the other hand, by Lemma 3-4
\[ 3-14 \quad A_{(\nu^\perp)^r} \eta = 0 \] for \( 0 \leq r \leq h-1 \).

By Codazzi's equation applied to the normal vector field
\[ \nu^\perp_{x_{h-1}} \ldots \nu^\perp_{x_1} \eta, \] we obtain
\[ 3-14 \quad A_{\nu^\perp_{x_{h-1}}} \ldots A_{\nu^\perp_{x_1}} Y = A_{\nu^\perp_{x_{h-1}}} \ldots A_{\nu^\perp_{x_1}} Z \] for all
\[ z, y, x_1,\ldots,x_h \in TM. \]

From Lemma 3-3 together with (3-10) and (3-12), it is clear that
\[ 3-15 \quad <H(p), (\nu^\perp)^h \eta(p)> = 0. \]

We shall show that (3-13), (3-15) and (3-16) imply that
\[ A_{(\nu^\perp)^h} \eta(p) = 0. \] Then, from (3-14) and Lemma 3-5, we obtain that
\[ \eta_p \in N_{h+1}(p) \] and the proposition follows.

Let \( z_1,\ldots,z_n \) be an orthonormal basis and \( i_0 \) a fixed index. From
Codazzi's equation applied to the normal vector field
\[ \delta = \nu^\perp_{x_{h-1}} \ldots \nu^\perp_{x_1} \eta, \] we obtain
\[ A_{\nu^\perp_{z_{i_0}}} Z_j = A_{\nu^\perp_{z_{i_0}}} Z_{i_0} \]
so that
Then
\[
\frac{1}{n} \sum_{j=1}^{n} \langle Z_j, Z_j \rangle \langle A^\perp Z_j, Z_j \rangle = \frac{1}{n} \sum_{j=1}^{n} \langle Z_j, Z_j \rangle \langle A^\perp Z_j, Z_j \rangle.
\]

Thus
\[
\frac{1}{n} \sum_{j=1}^{n} \langle Z_j, Z_j \rangle \langle A^\perp Z_j, Z_j \rangle = \frac{1}{n} \sum_{j=1}^{n} \langle Z_j, Z_j \rangle \langle A^\perp Z_j, Z_j \rangle.
\]

From (3-16), we obtain that, at \( p \)
\[
(3-17) \quad \frac{1}{n} \sum_{j=1}^{n} \langle Z_j, Z_j \rangle \langle A^\perp Z_j, Z_j \rangle = 0.
\]

If \( Y_1, \ldots, Y_n \) is a pseudo-orthonormal basis, put in (3-17):
\[
Z_1 = \frac{Y_1 - Y_2}{\sqrt{2}} \quad \text{and} \quad Z_j = Y_j \quad \text{for} \quad 3 \leq j \leq n.
\]

Then, it follows that
\[
(3-18) \quad 2 \langle A^\perp Y_1, Y_2 \rangle + \frac{1}{n} \sum_{j=3}^{n} \langle A^\perp Z_j, Z_j \rangle = 0.
\]

By Theorem 0.4 and Proposition 0.5 of [3], we need to consider four cases.

**CASE 1.** There exists a pseudo-orthonormal basis \( Y_1, \ldots, Y_n \) such that
\[
A^\perp = \begin{bmatrix}
\lambda_0^1 & 1 & & & \\
0 & \lambda_0^1 & 1 & & \\
& & \ddots & \ddots & \\
& & & \lambda_k^1 & 1
\end{bmatrix}, \quad A^\perp = \begin{bmatrix}
\lambda_0^j & b^j & -j & \cdots & c_k^j \\
0 & \lambda_0^j & 0 & \cdots & 0 \\
& & \ddots & \ddots & \\
& & & \ddots & \ddots \\
& & & & \ddots
\end{bmatrix}
\]

An easy computation shows that \([A^\perp_0, A^\perp_j] = 0\) for \( 1 \leq i, j \leq n \). Let us consider the positive definite subspace \( V = \text{span} \{Y_3, \ldots, Y_{k_0}\} \) and the symmetric linear transformations \( \bar{A}_i = V \rightarrow V \) for \( i = 3, \ldots, n \), defined by
Since $A_i^0 = \bar{A}_i$ for $1 \leq i \leq n$, thus there exists an orthonormal basis $\bar{Y}_1, \ldots, \bar{Y}_{k_0}$ of $V$ which diagonalizes simultaneously the matrices $\bar{A}_i$ for $1 \leq i \leq n$. The matrix $A_i^0$ does not change for the new basis $\bar{V}_1^\delta$. $Y_1, Y_2, \ldots, Y_{k_0}, Y_{k_0+1}, \ldots, Y_n$, and for the other matrices, we have (dropping the bar)

\[
A_{\bar{V}_1^\delta} = \begin{pmatrix}
\lambda_0 & b^j_0 & a^j_1 & \cdots & a^j_{k_0} \\
0 & \lambda_0 & 0 & \cdots & 0 \\
0 & a^j_1 & \gamma^j_1 & \iddots & \\
\vdots & \iddots & \ddots & \ddots & \\
0 & a^j_{k_0} & \gamma^j_{k_0} & \cdots & a^j_{k_0} \\
A_{\bar{V}_2} & \cdots & \cdots & \cdots & A_{\bar{V}_n}
\end{pmatrix}
\]

From (3-15), we obtain

i) for $Z = Y_1$ and $Y = Y_2$

\[\lambda_0^2 = 1.\]

ii) for $Z = Y_1$ and $Y = Y_j$, $3 \leq j \leq k_0$

\[\lambda_0^3 = \cdots = \lambda_{k_0}^o = 0.\]

iii) for $Z = Y_2$ and $Y = Y_j$, $3 \leq j \leq k_0$

\[a^j_j = \gamma^2_j.\]

From (3-18), we obtain

iv) $\gamma^j_j = 0$, $3 \leq j \leq k_0$.

v) $Z + \sum_{j=3}^{k_0} a^j_j = 0$.

From (3-16), we have that

\[
A_{\bar{V}_1^\delta} A_{\bar{V}_2^\delta} A_{\bar{V}_j^\delta} A_{\bar{V}_2^\delta} = A_{\bar{V}_1^\delta} A_{\bar{V}_j^\delta}, \quad 3 \leq j \leq n.
\]

In particular, comparing the $j^{th}$ element of the first line of both
matrix products, we obtain

\[ \lambda^2 a_j + a_j^2 \gamma_j = \lambda^2_0 a_j + \gamma_j^2 a_j, \quad 3 \leq j \leq k_0. \]

From i), ii), iii), iv) and vi), it follows that

\[ a_j^2 = (a_j^2)^2, \]

which is a contradiction to v). So case 1 is not possible.

**CASE 2.** There exists a pseudo-orthonormal basis \( Y_1, \ldots, Y_n \) such that

From (3.15), we have

\[ A_{Y_1}^\perp = A_{Y_2}^\perp, \]

Thus

\[ \lambda_0^1 Y_2 + Y_3 = \mu^2 Y_1, \]

which is not possible. So Case 2 cannot occur.

**CASE 3.** There exists an orthonormal basis \( Y_1, \ldots, Y_n \) such that

From (3.15), we obtain

i) for \( Z = Y_1 \) and \( Y = Y_2 \)
\[ \beta_1 = \alpha_2 \]

From (3-17), we obtain

\[ \beta_1 = -\alpha_2 \]

Then \( \beta_1 = 0 \), which is a contradiction. So case 3 is not possible.

CASE 4. There exists an orthonormal basis \( Y_1, \ldots, Y_n \) such that

\[
\begin{pmatrix}
\lambda_0 I_{k_0} \\
\vdots \\
\lambda_{k_j} I_{k_{j+1}} \\
\vdots \\
\lambda_{k_2} I_{k_2}
\end{pmatrix}
\]

It is easy to see that the basis can be chosen in such a way that

\[
\begin{pmatrix}
A^j \\
Y^j_{k_0 + 1} \\
\vdots \\
Y^j_{k_j} \\
Y^j_{k_2}
\end{pmatrix}
\]

for \( 2 \leq j \leq n \)

where \( A^j = (a_{k_j}^j) \) is a \( k_0 \times k_0 \) matrix.

From (3-15), it follows that

\[ \gamma_s^t = 0 \quad \text{if} \quad s \neq t \quad \text{and} \quad k_o + 1 \leq s, t \leq n . \]

Then (3-17) implies that

\[ \gamma_s^s = 0 \quad \text{for} \quad k_o + 1 \leq s \leq n . \]

Using (3-15) for \( Z = Y_j, \quad 1 \leq j \leq k_o \), and \( Y = Y_t, \quad k_o + 1 \leq t \leq n \), we obtain

\[ A^j_{Y_t} = 0 \quad \text{and} \quad \gamma^j_{Y_t} = 0 . \]

Then, from (3-17), it follows that

\[ \sum_{j=1}^{k_o} \langle Y_j, Y_j \rangle_A Y_j = 0 . \]

Thus
So we obtain
\[ -\lambda_0 + (k_0 - 1) \lambda_0 = 0. \]
Therefore, \( \lambda_0 = 0 \) or \( k_0 = 2 \).

If \( \lambda_0 = 0 \), from (3-15) for \( Z = Y_1 \) and \( Y = Y_k \), \( 1 \leq k \leq k_0 \), we obtain
\[ a_{11}^k = \ldots = a_{1n}^k = 0. \]
But then, all the matrices can be simultaneously diagonalized and the same argument as in the beginning of this case shows that they must vanish.

If \( k_0 = 2 \), we have
\[
\begin{align*}
A_{Y_1} & = \begin{pmatrix} \lambda_0 & 0 \\ 0 & \lambda_0 \end{pmatrix}, \\
A_{Y_2} & = \begin{pmatrix} a_{11} & a_{12} \\ -a_{12} & a_{22} \end{pmatrix}
\end{align*}
\]
From (3-15), we obtain
\[ \lambda_0 = -a_{12}, \quad a_{11} = 0. \]
From (3-17), we obtain
\[ \lambda_0 = a_{12}, \quad a_{22} = 0. \]
Thus
\[ A_{Y_1} = 0 = A_{Y_2}, \]
which concludes the proof.

PROOF OF THEOREM 1-1. From Proposition 3-2, we have
\[ N_{m+1}^\perp(p) = \{ \xi \in N_m^\perp(p) : ((\nabla^\perp)^m R^\perp)(\xi) = 0 \quad \text{and} \quad (\nabla^\perp)^m H(p) \perp \xi \}. \]
Thus
\[ N_m^\perp(p) = N_m^\perp(p) \text{ if and only if } \begin{cases} (\nabla^\perp)^m R^\perp|_{N_m^\perp} = 0 \\ (\nabla^\perp)^m H \subset N_m^\perp. \end{cases} \]
It follows that \( N_m \) is a parallel normal subbundle and we can apply Proposition 3.2 to complete the proof.

REMARK. In fact, we proved that
N_m is parallel if and only if
\[ \left\{ \begin{array}{l}
(V^+)^m R^+|_{N_m} = 0 \\
(V^+)^m H \subset N_m.
\end{array} \right. \]

REMARK. The Theorem 1-1 remains valid if the immersion is m-regular in an open connected and dense subset of \( M^n \).

REFERENCES

