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THE 16TH CENTURY IBERIAN CALCULATORES

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This article is an extension of the letter I wrote to the Editor of the American Mathematical Monthly and which appeared in the April 1990 issue. Some people have written to me asking for a contributed article on the subject, others, have expressed great interest and requested further information on the 16th century Spanish Mathematics and Physics. It seems quite natural to include in this volume dedicated to Rey Pastor a follow up to his landmark book "Los matemáticos españoles del siglo XVI" [7]. The task is not easy to accomplish in a few pages, therefore, I will skip many details and discuss only a few central facts concerning Mathematics and Mechanics as it was cultivated by the Spanish and Portuguese Algebraists and "Calculatores"[1] of the time. However, the main emphasis will be on the "Calculatores" rather than on the "Algebraists". The rationale behind that choice is a natural one, for the work of the Iberian "Calculatores" contained the seeds that ultimately germinated in the development of modern Mechanics, the science shaped by Galileo during the the first half of the 17th. century (this thesis was initially proposed by P. Duhem in 1913, [3] and has been upheld since by many historians of science, among them, Wallace, [8], [9], [10]). I will consider here also the insertion of the Iberian school of calculatores as a segment of one of the branches of the tree that constitutes History of Calculus.

It should be pointed out that early historians of Mathematics have had an incomplete view of the impact that English and French Mechanics had on Mathematics over the period 1350-1600. This issue is discussed below.

Concerning the development of Algebra in Spain during XVI century, I will focus on two names associated with highly original contributions. The first, Fray Juan de Ortega, who was a Dominican priest (also a renowned arithmetician), whose approximation meth-

[1] The term "Calculator" was used to describe the mathematician or physicist doing research in Motion Theory, involved with numerical examples, and using as main tools the elements of Proportion Theory and Infinitesimal Arithmetic. An interesting definition has been provided by Wieleitner in [11], p.166: "Calculator" war ein Gelehrter, der sich besonders mit der Untersuchung des "Motus" befasste, wo natürlich immer, wenn auch meist einfache, Zahlenbeispiele benützt wurden.

ods for square and cube roots have been widely commented in [1], [4] and mainly by Rey Pastor in [7], who shows a real grasp and understanding of Ortega's method[2]. Second is the Portuguese mathematician Pedro Núñez (Nonius), whose celebrated solution to the problem of the loxodrómic curve made him famous in a world concerned with scientific navigation. Núñez, who was a professor at Salamanca, Spain, and at Coimbra, Portugal, authored a celebrated treatise on Algebra, written in Spanish, that contained many of the original views on the modern Algebra of the time ("Livro de Algebra en Arithmetica e Geometria", Antwerp 1567, among many known European editions,). An acurate analysis of Pedro Núñez's mathematical output can be consulted in [7], also [1], II, p. 358. Rey Pastor in [7] completes the list of algebraists by adding the name of Marc Aurel, a German scholar whose main contribution was to introduce modern Algebra to Spain, circa 1552, ("Libro primero de Aritmética Algebratica", Valencia, 1552). Here, I refer to the "rule of the Cosa" or "Die Coss" as modern Algebra, as developed in Italy and in Germany around 1520. Other algebraists mentioned by Rey Pastor in [7] are: Joan Pérez de Moya and Antich Rocha.

Returning to the field of Infinitesimal Arithmetic and Mechanics, an area in which the Iberian mathematicians excelled during the XVI century, the two names that are at the forefront of the Iberian school of Paris-Salamanca-Alcalá de Henares are Alvaro Thomás and Fray Domingo de Soto, [3], [8], [9]. According to Pierre Duhem, [3] pp 263-562, "Dominique de Soto et la escolastique parisienne", the scientific works of the so-called Spanish school of Paris was the step that allowed the early Mechanics of the masters of Merton College (Oxford, circa 1340), Thomas Bradwardine, Suiseth (Richard Swineshead) and William Heytesbury, to mature into the views of Galileo and Galileo's Science. Domingo de Soto has become a key figure in History of Science because of his early adumbration of the law of falling bodies and his central role in the Spanish school of Parisian Calculatores. With reference to this particular point, see [3], [8] and [9]. For a more complete view on the Merton College Mechanics the reader is advised to consult [2].

The Iberian scientific forebearers of Galileo

The Spanish school of Parisian Calculatores can be traced to the Scot John Maior, 1469-1550, logician, mathematician, natural historian and philosopher, who spent most of his productive life in Paris where he formed a school of philosophers whose influence was unparalleled in his time. His teachers were the Spanish logician Geronymo Pardo, Thomas Bricot and the Scotist Peter Tartaret. The dialectic of the time between nominalism and realism, (Kinematics versus Dynamics as the essence and nature of motion[3]), found him in a position almost eclectic in spite of his nominalist preferences. As we are going to see,

[2] As Rey Pastor points in out in [7] p.p. 72-81 M.Cantor and G.Eneström completely missed the point in understanding Ortega's method. Rey Pastor goes on to explain it with great detail and care showing its connection with the famous Pell equation. In fact, Ortega's method compares favorably with the ones developed by Heron and Bombelli. As Rey Pastor says: "El método de Ortega es distinto de ambos y es mejor", p.78. The work of Ortega was widely known in Europe, "Tratado subtilíssimo de Arismética y Geometría", 1512, 1515, 1522, 1534, 1537, 1542, 1552, 1563 and 1612, are the main editions.

[3] As Clagett points out [2], p.207 : "Put in somewhat modern terminology, this was

this fact was to have an impact on the eclectic views of the Iberian Masters. Maior was an avenue through which the Mechanics of the Mertonians infuenced the 16th century Parisian Calculatores and ultimately reached Galileo.

Among the most famous students of John Maior we find the Flemish Calculator Jean Dullaert of Ghent, the Spaniard Gaspar Lax de Sariñena (the first Iberian Calculator), the Portuguese prodigy Alvaro Thomas and the Spanish brothers Luis and Antonio Núñez Coronel. All of them reached great fame and success as professors in the colleges of the University of Paris circa 1500 [10]. Among the students of Gaspar Lax we find the Spaniard Juan de Celaya, whom I shall refer to below as a sequel to Alvaro Thomás and as an introduction to Soto, Celaya's most famous student. Domingo de Soto, in turn, would become the center of this remarkable school which has been described by Duhem, quoting Louis Vives, as "les plus subtils, les plus abstrus disputeurs de l'Université de Paris..." at the beginnng of the 16th century, [3] p.531. Later, Soto would form a school of philosophers and physicists among whom we find at least one of Galileo's professors, Francisco Toledo [8], [10]. The Iberian influence on Galileo was exercized mainly by his Jesuit professors at the Collegio Romano. Aside Soto's student, Francisco Toledo, we find the Spaniard Benito Pereiro, a product of the Spanish school of Calculatores. Among the non-Iberian members of that school, we find the German Christopher Clavius, a celebrated mathematician, student of Pedro Núñez at Coimbra, who would have a ponderous impact on Galileo's early years, [10] p.p. 310, 311.

Gaspar Lax de Sariñena

Gaspar Lax de Sariñena was born in the kingdom of Aragón, Spain, in 1487 and died in Zaragoza in 1560. As said before, he was a disciple of John Maior at Montaigu and professor of Arts in that college of Paris much later. He had the reputation of having been a better mathematician than his teacher John Maior. He was called by the people that knew him as "the prince of the Parisian disputeurs". He wrote many books while being in Paris, and later in Spain, he wrote a treatise on Logic and another one on Physics. Perhaps the most remarkable of his mathematical works was his "Arithmetica speculativa Magistri Gasparis Lax aragonensis de Sarinyena duodecim libris demonstrata", Paris 1515. Other mathematical works written in Paris were: "Proportiones Magistri Gasparis Lax aragonensis de Sarinyena", (1515) and "De propositionibus arithmeticis", 1515. He, later published in Zaragoza "Questiones physicales", 1527. Among his disciples we find the Portuguese Francisco de Mello and the celebrated Juan de Celaya, whom we shall refer to after Alvaro Thomas.

Ciruelo and Silíceo

The Master Pedro Sanchez Ciruelo, (1470-1554), was born in Daroca, Aragón, edu-

an argument between those who would define movement primarily in terms of forces and those who would define it kinematically. The argument had come to the fore in the commentary on the Physics of Aristotle by the Spanish Moslem philosopher Averroes. He argued for the dynamic definition against the kinematic approach of his fellow Spaniard Avempace."

cated in Salamanca and later got his doctorate in Paris, where he became a professor of mathematics. In 1516 Ciruelo returned to Spain to teach at Alcala. During the same year he published his "Cursus mathematicarum artium liberalium", but, he is better known for his Parisian publications: "Thomae Bradwardini Arithmetica speculativa ex libris Euclidis, Boecii et aliorum revisa et correcta a Petro Sanchez Arragonensi", 1496, 1503. "Tractatus Arithmeticae practicae", 1496, 1505, 1509, 1514. "Thomae Bradwardini Geometria speculativa recolligens omnes conclusiones geometricas, studentibus Artium et philosophiae Aristotelis valde necessarias, simul cum quodam tractatu de quadratura circuli, noviter edito", 1496, 1502, 1512. "Commentarius in Spheram Joannis de Sacro Bosco cum Petri de Alliaco in eundem questionibus", 1499, 1505, 1508.

Juan Martínez Silíceo (1486-1557) was born in Villagarcía, Extremadura, Spain. He was educated at the University of Paris, where later was to became a professor. Silíceo was known to have been a disciple of the Scot Robert Caubraith, who was a professor at the Coqueret College. Silíceo's association with the school of John Mayor was through John Dullaert, who was his teacher at the College of Beauvais. Among his most important works, we find: "Ars Arithmetica Martini Silicei: in Theoricam et Praxim scissa", Paris 1514. "Calculatoris Suisset anglici sublime et prope divinum opus in lucem recenter emissum: a multis quibus ante hac conspersum fuerat mendis expiatum, et novis compendiosisque titulis illustratum, novo tandem ordine quo lucidus foret digestum atque distinctum: cura atque diligentia philosophi Silicei", Salamanca, 1524, better known by the title "Liber calculationum". This last work, as the title suggests, was concerned mainly with the Mertonian Calculus. Silíceo once back in Spain, achieved the highest of the honors: Archbishop of Toledo, Cardinal and personal tutor of Felipe II.

Alvaro Thomás

Alvaro Thomás was born in Lisbon, probably during the second half of the 15th century. Nothing is known about his childhood or his family. We find Alvaro at the beginning of the 16th century in Paris as Rector of the Coqueret College, as an accomplished scholar and author. The work of this remarkable mathematician is known through a famous treatise published in Paris in 1509 titled "Book on the Three Kinds of Movement, With Ratios Added, Explaining in Part Swineshead's Philosophical Calculations"[4]. As Duhem has pointed out, the object of the treatise was to elucidate the calculations as well as to point out the errors of the "Liber Calculationum" by Richard Swineshead [3], p.532. The book, however went far beyond this task, because it presented the reader with an eclectic view of the advances of Merton's Mechanics and a thorough discussion of the calculatory methods associated with the subject. In spite of being a physist of considerable ability, his main strength appeared to have been Mathematics, where he showed an impressive originality and independence of judgement.

One of the main accomplishment of the Merton scholars was the celebrated Theorem

[4] The title in Latin reads "Liber de Triplici Motu Proportionibus Annexis Magistri Alvari Thome Ulixbonensis Philosophicas Suiseth Calculationes ex Parte Declarans". It consists of 162 unnumbered folia in double column and text written in gothic characters. The copy consulted by the author of these notes is a microfilm of the original at the University of Michigan.

of Uniform Acceleration, which in modern terms gives a measurement of the acceleration in terms of its medial velocity. The Mertonian term for a motion uniformly accelerated was "motus uniformiter difformis" and it referred to the variation of the velocity. "Motus uniformis" meant uniform motion (constant velocity). "Motus difformis" meant non uniform motion and it was classified as "motus uniformiter difformis" and "motus difformiter difformis"; the former is dicussed above, and the latter stood for what today is described as a motion with non constant acceleration. Here, I have considered implicitly that the variation of the motion is measured with respect to time (studied by Alvaro under the heading "De motu locali quo ad effectum tempore difformi' "). When the variation is with respect to the subject (for instance, the rotation of a wheel, where different points have different velocity) the latin heading used by Alvaro was "De motu locali quo ad effectum subiecto difformi' ".

In the case of a "motus difformis" one of the big questions at the time was how it can be measured. Different scholars proposed different answers, among them, the choice was the maximum velocity within the time interval or within the subject. William Heytesbury addressed this point by proposing the reduction to uniformity, in the case of a motion which difforms with respect to time, that is, finding what we could call a Mean Velocity. Heytesbury however, suggested the maximum velocity as the way of measuring a motion that difforms with respect to the subject (Later we shall discuss the comments that Juan de Celaya had to make on this very same issue). Alvaro would contest Heytesbury's use of the maximum velocity in the second instance, making a strong case for the use of the mean velocity instead.

The natural question that arises is whether such a Mean Velocity exits for any given motion; and, if the answer is affirmative, how can this Mean Velocity be evaluated. The reader will notice the intimate connection that these questions have with the mean value theorem, either the differential or the integral versions. Alvaro goes on to consider both questions.

Alvaro gives a thoughtful answer to the first question: "In spite of the objections, it is a common opinion of local philosophers, an opinion with much vigor and force, that while a difformis motion takes place, any general one, a certain space is negotiated. On the other hand, the same space can be negotiated in the same time with the help of an uniform velocity"[5]. In other words, Alvaro gives an intuitive but powerful answer to the existence of such a Mean Velocity as envisaged by William Heytesbury.

Concerning the second question, the possibility of a "difformiter difformis" nature of the velocity, led Suiseth and later Oresme to consider ideal motions over a time interval

[5] In Alvaro's own words: "In oppositum tamen est universalis opinio communiter philosophantium que in hac parte multum vigoris ac roboris habet. Preterea per quemlibet talem motum difformen in totali tempore adequate pertransitur aliquod spacium adequate: et tale spacium in tali tempore ab aliqua velocitate uniformi natum est pertransiri: igitur illa velocitas uniformis est tanta quanta est velocitas illius motus uniformis quo illud spacium in eodem tempore pertransitur adequate". The quotation comes from "Liber de Triplici Motu", second treatise, third chapter, six pages after the beginning of the chapter, second column.

T, subdivided as:

$$T = T_1 + T_2 + T_3 \cdots \qquad T_k = x^k; 0 < x < 1$$

being $x = \frac{1}{2}$ or $x = \frac{1}{4}$ as the most usual choices.

The velocity was thought on each time interval T_k as taking the value:

$$V = v_k \text{ over } T_k,$$

(the example considered by Oresme was $v_k = k$ and $x = \frac{1}{2}$).

Hence, the measure of the total space traveled by the subject led to the summation of the series:

$$v_1 x + v_2 x^2 + v_3 x^3 + \cdots$$

The evaluation of the "Mean Velocity" required to sum the above series. Alvaro considered many types of series that he managed to reduce to the geometric case after some skillful manipulations. In some other cases he indicated their divergence (for instance when the series could be reduced to an equivalent geometric one of ratio $x > 1$, "proportio superparticularis"), ref [11].

In some cases where he could not come up with the sum, he indicated two bounds for the sum, one from below and the other one from above. In this approach he departed from all the earlier Calculatores. This technique appeared for the first time in Alvaro's work and constitutes one of his most original contributions. When our prodigy could not successfully sum the series, he resigned himself to the approximation as an answer[6].

Finally, Alvaro in spite of not being able to find the exact numerical value of the "Mean Velocity" he believes in its existence as indicated by his intuitive proof of the "Mean Velocity" theorem. [7] Let us examine some of the basic ideas in Alvaro's treatment of numerical series in context of motion theory. Nicole Oresme in his treatise "De difformitate qualitatum" see [3], p.393, considers the series :

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots$$

[6] In Alvaro,s own words: "Impossibile est naturaliter intellectum finite capacitatis talem velocitatem sic difformen ad uniformitatem redigere et adequatum spacium pertransitum infallibiliter assignare". Here, Alvaro blames the boundedness of the natural intelligence for not being able to assigne a precise value to the sum giving the reduction to uniformity.

[7] In fact, he believes that after death the incognita will be resolved: "Credo tamen animas separatas a corpore et intelligentias in imperspecto tempore omnia ista cognoscere. Cesset igitur dolor querulantium et non putat homo sua terminus...". This quotation, as well as the previous one, come from the same column in the 3rd chapter, 2nd Tractatus, Liber de Triplici Motu.

and finds a geometric method to sum it.

Alvaro generalizes Oresme,s result to series of the form, (see [11]), :

$$\frac{c}{f}\left(1+\frac{2}{g}+\frac{3}{g^2}+\cdots\right)=\left(\frac{c}{f}\right)f^2$$

Where $f=(1-\frac{1}{g})^{-1}, g>1$

Alvaro's proof, although rethorical, can be expressed in modern terms as the double series (in which each row is a geometric series, whose sum constitutes also a geometric series):

$$\begin{aligned}1+\frac{1}{g}+\frac{1}{g^2}+\frac{1}{g^3}+\cdots &= \left(1-\frac{1}{g}\right)^{-1}\\ \frac{1}{g}+\frac{1}{g^2}+\frac{1}{g^3}+\cdots &= \frac{1}{g}\left(1-\frac{1}{g}\right)^{-1}\\ \frac{1}{g^2}+\frac{1}{g^3}+\cdots &= \frac{1}{g^2}\left(1-\frac{1}{g}\right)^{-1}\\ \cdots &= \cdots\end{aligned}$$

Finally, let us consider a remarkable series for which Alvaro gives upper and lower estimates for its sum :

$$1+\frac{7}{4}.\frac{3}{5}+\frac{7}{4}.\frac{11}{8}.\left(\frac{3}{5}\right)^2+\frac{7}{4}.\frac{11}{8}.\frac{19}{16}.(\frac{3}{5})^3+\cdots$$

The lower estimate given by Alvaro is $2+\frac{1}{2}$ and the upper estimate is $6+\frac{1}{4}$. He gives no direct proof for these estimates, but refers to a similar result treated shortly before as Correlarium 1. The latin text reads :

"Sequitur secundo que hora divisa per partes proportionales, proportione superbipartiente tertias: mobili moto in prima parte proportionali aliquantula velocitate: et in secunda in proportione super tripartiente quartas velocius: et in tertia in proportione super tripartiente octavas velocius quam in secunda: et in quarta in proportione supra tripartiente octavas velocius quam in tertia: et sic consequenter spacium pertransitum in totali hora erit majus quam duplum sexquialterum ad spacium pertransitum in prima parte proportionali et minus quam sexdecuplum sexquiquartum".

Juan de Celaya

Juan de Celaya was born in Valencia, Spain, circa 1490 and died in Turia, Spain, in 1558. His father was a gentleman (caballero hijodalgo) that participated in the Granada war. It is believed that he initiated his education at the University of Valencia. We see him later in Paris, circa 1500, at the Montaigu College, where he studied under the Spaniard Gaspar Lax and the Flemish Master John Dullaert, both were, as said before, students of John Maior. He completed studies in Theology, earning his doctorate in 1522. He was a gifted mathematician and physicist. Between the years 1510 and 1515, he taught at the Coqueret College with his associates Alvaro Thomás and the Scot John Caubraith. The former had a decisive influence on Celaya's work, specially on the mathematical part. The latter would become Silíceo's mentor. In 1515 he transfered to the College of Sainte Barbe, where he was a professor until 1524. This period is particularly important because he counted among his disciples the Segovian Domingo de Soto and the Portuguese Juan Ribeyro. The former would become the first scientist in adumbrating the law of falling bodies.

Of Celaya's teachers, Gaspar Lax was influential on his formation in Mathematics. Later in Spain, Celaya was at the center of a school that counted among its members a Pedro Encinas, Pedro Ciruelo, Juan Martínez Silíceo and the Dominican Juan the Ortega, who collectively produced most of the mathematics textbooks used in most European universities at the time, [10] p.80. Celaya returned to Spain in 1524, where he became Rector in "perpetuo" of the University of Valencia.

Celaya comments on the "Mean Velocity", following very much the views we have discussed when referring to Thomás [8]. Like Thomás, Celaya deals with the Physico-Mathematics of Swineshead, Heytesbury and Oresme, and his advance over these authors is mainly his systematization; but in certain issues he showed much more imagination[9]. Celaya's treatment of numerical series is very much influenced by Alvaro. We see in his book "Expositio in octo libros physicorum Aristotelis cum questiones eiusdem", Paris

[8] On his commentaries on "Physics, De Coelo and De Generatione" (copy at the University of Chicago), he refers to Jacobus de Forlivio's views on how to measure a difformis motion "...est opinio: una est Jacobi de Forlivio asserentis velocitatem in talibus metiri penes maximum gradum velocitatis sicut dicetur de intensione qualitatis difformis et eadem argumenta vel similem difficultatem habentia possunt fieri." Here, as said before, when discussing Thomás, the possibilty of measuring a difformis motion through its maximum velocity was considered by Forlivio. Celaya continues below: "Alia est opinio Guillelmi Hentisberi et calculatoris et fere omnium aliorum philosophorum tenentium que in tali motu difformi quo ad tempus opportet (?) reducere difformitatis ad uniformitatem et penes talem gradum ad quem reducentur velocitas debet mensurari". Here, Celaya refers to the opinions of William Heytesbury and Calculator (Suiseth), that the motion should be reduced to uniformity, and measure it with respect to that value (grado), that of the "mean velocity" ! These quotations come from the Tertius Liber Phisicorum, folium 58 vta, opus cited above.

[9] Celaya thought that Mathematics could be applied to Medicine, and even to Theology; all we have to do is to replace the terms "to move" by "to become feverish" or "to merit", [10] p.81. The original comes from op. cit. "Tertius Liber Phisicorum", Folium 88: "Hys prelibatis ponuntur alique conclusiones: quas Bernardus tornius florentinus commentator Hentisberi Nicolao orem ascribit. Nam ipse invenit fundamentum teste enim philosopho... ...Conclusiones non solum ad medicinam, verum ad sacram theologiam applicari valent mutando illum terminum moveri vel motus in aliquem istorum terminorum, scilicet, febris vel meritum vel mereri."

1517, third book (under the heading: "De motu difformi quo ad tempus") folium 88, second column, the series:

$$1 + 2x + 3x^2 + 4x^3 + \cdots = (1-x)^{-2}$$

Where T (total time) is given by the series :

$$1 + x + x^2 + x^3 + \cdots = (1-x)^{-1}$$

Here, we have chosen the first proportional part to be equal to 1. Celaya expresses the mean velocity through T is in the same proportion to the velocity in the first proportional part as T is to 1. Celaya's Latin reads:

"Prima igitur conclusio est ista: diviso aliquo corpore quavis proportione per infinitas partes proportionales si aliquod mobile moveatur in prima illarum aliqualiter velociter: et in secunda in duplo velocius: et in tertia in triplo velocius quam in prima: et in quarta quadruplo velocius quam in prima: et sic consequenter ascendendo per omnes species proportiones multiplicis tale mobile in tota hora movebitur velocius quam in prima parte proportionali in ea proportione qua se habet totum id tempus ad primam eius partem proportionalem dividendo illo modo: ita que si totum id tempus se habet in proportione dupla ad primam partem proportionalem tale mobile in duplo velocius movebitur in toto illo tempore...".

Here, Celaya, following Alvaro, expresses the "mean velocity" in terms of the velocity in the first propotional part. This establishes a relation between the sum of the series and the first term. When that relationship can not be established, Celaya, as Alvaro did before, gives upper and lower estimates for the sum.

Let us illustrate Celaya's handling of the series (op. cit f 90):

$$S = 1 + \frac{2}{1}.\frac{1}{2} + \frac{3}{2}.\frac{1}{2^2} + \frac{4}{3}.\frac{1}{2^3} + \cdots$$

Celaya gives the following estimates for S:

$$2 < S < 4$$

For the proof, he considers first an uniform motion, that has a fixed constant velocity on each proportional part. For that motion the total space traveled is twice that of the first proportional part. This motion is slower on each proportional part than the given motion; therefore the lower estimate holds truth. In order to show the upper bound Celaya indicates that the given motion is slower on each proportional part than the one:

$$1 + 2.\frac{1}{2} + 3.\frac{1}{2^2} + 4.\frac{1}{2^3} + \cdots$$

that adds up to 4. Celaya's text reads:

"Decima conclusio est ista. diviso aliquo tempore per partes proportionales proportione dupla: si fortes in prima parte proportionali moveretur aliqualiter velociter in secunda in duplo velocius quam in prima et in tertia in sexquialtero velocius quam in prima et in quarta sexquitertio velocius quam in prima: et sic consequenter procedendo per omnes species proportiones superparticularis spacium pertransitum in toto illo tempore est maius quam duplum ad spacium pertransitum in prima parte proportionali et minus quam quadruplum..."

Domingo de Soto

Domingo de Soto was born in Segovia, Spain circa 1494 and died in Salamanca, Spain, 1560. His original name was Francisco, he later changed it to Domingo, at the moment of his ordination as Dominican priest (1525). Soto received his Latin training at Segovia and later transfered to the University of Alcalá where he studied logic and natural philosophy, receiving his "Bachiller" title in 1516. We see him shortly after in Paris, studying under Juan de Celaya, in the Santa Barbara College, where he received a Master of Arts degree. Later, he came under the influence of John Maior, who was teaching at the Montaigu College together with the Spanish Masters Luis and Antonio Núñez Coronel. Another personality who had great influence on the young Soto was the famous Spanish Thomist Francisco de Vitoria, who at the time was lecturing at the Dominican priory of Saint-Jaques. In 1519 Soto returned to Spain to teach at Alcalá. From that point on, Soto's commitment to the Church became deeper. Theology and Philosophy were to be his main foci of interest. As said before, he was ordained priest in 1525. Among his works, the one that is of interest to us in this paper is his commentary and questions on the Physics of Aristotle ("Super octo libros physicorum Aristotelis subtilissime questiones", 1545, 1551, 1555).

There are two main contributions made by Soto to Physico-Mathematics. The one in which he describes the motion of the falling bodies as uniformly accelerated or, in his own words, "uniformiter difformis"[10]. The other one is concerned with a very original and efficient systematization of the study of motion as pointed out by Wallace in [9]. Wallace, quoting Alexander Koyré, brings up the "enigma" of why was Soto alone describing falling bodies as a uniformely accelerated motion ? and why did no one else before Galileo... adopt those views ?. He answers the ridle in a series of papers the first of which is [9]. The plan in Wallace's research is a deep comparative study of the teachings of Soto's predecessors and contemporaries. Here, is where Soto's systematization and simplifications play a crutial role: among all the authors studied by Wallace, he (Soto) and the Spanish calculator Diego Diest presented the views and exemplifications in which

[10] See [10] p.109 where this famous citation can be consulted. Soto's words are: "Hec motus species proprie accidit naturaliter et proiectis. Ubi enim moles ab alto cadit per medium uniforme, velocius movetur in fine quam in principio. Proiectorum vero motus remissior est in fine quam in principio; atque adeo primus uniformiter difformiter intenditur, secundus vero uniformiter difformiter remittitur". Here, Soto refers to the motion of free fall and the motion of a projectile launched upward in an uniform medium. The first one is uniformly accelerated, the second one uniformly decelerated.

a precise quantitative modality could be assigned to falling bodies.

There is still another role played by Soto. It concerns the transmission of the science of the Mertonian and Parisian Masters to the science of Galileo. The central figure in that transmission is Soto. This thesis was cast initially by Duhem [3]. The profound historical analysis made recently by Wallace [10] [11], tends to confirm that hypothesis and to show other original views, namely, the impact of the Collegio Romano on Galileo's scientific and philosophical formation (in particular the Thomist views). His student Francisco Toledo became one of Galileo's professors at the Collegio Romano. As commented before, Toledo was not the only Iberian calculator who taught Galileo, Benito Pereyro was another member of that remarkable school. Among the non Iberian masters that perfected their studies in the Iberian peninsula, we find the celebrated German scholar Christopher Clavius, (1537-1612), disciple of Pedro Núñez at Coimbra (Portugal). Clavius was a professor at the Collegio Romano and one of Galileo's most famous mentors. Among Clavius students, aside from Galileo, Gregoire de Saint Vincent, (1584-1667), was probably the most accomplished one, as a key figure in the history of Calculus. Likewise, Bonaventura Cavalieri (1598-1647), celebrated disciple of Galileo, (whose method of the indivisibles constituted a giant step toward modern integration theory), can be considered as a byproduct of this remarkable school.

Discussion

A great deal has been said about the presence of a substantial number of Iberian scholars at the University of Paris during the end of the 15th and beginning of the 16th centuries. It should be remarked here, that the impresive work of R. G. Villoslada "La Universidad de París durante los estudios de Francisco de Vitoria", 1938, Analecta Gregoriana 14, sheds light on this particular issue. It gives a very detailed account of the Iberian scholars at the University of Paris over that period of time. It shows that in most of the cases, their motivation for being there was for improving their theological education. Their training in Physics and Philosophy was a byproduct of that main objective. Rey Pastor in [7] shows similar views.

Coincidentally, this was the time when Spain opened up to the world and initiated a sequence of land discoveries and colonization. Together with this impressive development we see the surge of the Spanish science, notably that of the natural sciences exemplified by the works of Don Gonzalo Hernández de Oviedo y Valdez, Miguel Serveto and Fray Joseph de Acosta[12]. The 16th century was to witness the return of the Spanish scholars from their professorial chairs in Paris. Among the many scientists that returned to Spain

[11] See also "Galileo and his Sources", Princeton University Press, 1984.

[12] Oviedo wrote "De la natural hystoria de las Indias" and Acosta wrote "Historia Natural y Moral de las Indias". Both treatises have been very favorably commented by Humboldt in his "Cosmos". It is oportune to transcribe here some of the comments that Humboldt writes on Acosta's work: "The ground work of what we, at present, term as Physical Geography, independent from the mathematical considerations, is contained in the Jesuit Joseph de Acosta's work: Historia Natural y Moral de las Indias". Miguel Serveto is credited with the discovery of the pulmonary blood circulation. For a translation of Serveto's work see "Michael Servetus" by Charles D. O'Malley, 1953, Philadelphia.

to lecture at Alcalá and Salamanca we find: Celaya, Soto, Coronel and Ortega.

Once back in Spain, the school of Iberian Calculatores developed a scientific movement that projected itself to Italy and America. The repercusion of this movement in Italy was going to be felt in the Collegio Romano, and ultimately, it would reach Galileo [10]. Regarding the New World, one of Soto's students, Fray Alonso de la Vera Cruz, the father of Mexican philosophy, wrote "Physica Speculatio" in 1557 in Mexico, intended to be used at the University of Mexico (founded in 1553). This treatise was inspired by the work of Soto and constitutes an early American contribution to the calculatory systematization of Physics.

The very limited view that M. Cantor, [1], has had on the reception and on the spread of the English and French Physics and its influence on Mathematics had a negative influence on many historians of Mathematics in Spain [13]. Likewise, G. Eneström has shown similar views[14]. The works of Duhem [3], Wieleitner [11], and Wallace [8], [9] and [10] have put in perspective the real scientific importance of the Spanish Physico-Mathematics of the 16th century. The work that the Spanish Calculatores collectively did, consisted in the systematization and "modernization" of the Physics of the Mertonian Masters and the Parisian Doctores. The systematization was superbly achieved by Celaya y Soto, to mention only the most successful ones. In Soto's case that systematization allowed him to see the falling bodies as an uniformely accelerated motion. The deep historical and philosophical analysis made by Wallace in [9] and [10] permits to see Soto's simplification in its real perspective, the answer to "Soto's enigma". The "modernization" consisted in presenting an eclectic view between the two extreme positions, nominalist versus realist, thus adumbrating the interplay between Kinematics and Dynamics, whose integration occured almost a century later. The works of Soto contained another important novelty: very precise natural phenomena were brought into discussion and described properly. Finally, Thomás, virtuoso Calculator and subtle mathematician, perfected and refined the Calculus developed by the Mertonians and Nicholas Oresme. Particularly important was his contribution to the theory of numerical series in the context of Motion Theory.

This collective effort left the Physics of the 16th century Calculatores maximally systematized and ready for a successful next step, the science of Galileo.

Finally, one may ask what was the influence in Latin America of this surge of the

[13] Surprisingly enough, Rey Pastor himself refers to the 16th century mathematicians, in particular to the arithmeticians, "Calculatores", "sus obras nacieron con un pecado original, el no ser modernas,..." [7], p.41. Here, Rey Pastor, possibly influenced by the views of Cantor, refers to the backward character of the English-French-Spanish mathematics of the 16th century calculatores. The works of Duhem, Wieleitner and Wallace were to show this statement inacurate.

[14] G.Eneström in [4] has asked; " Quels ont été les mérites scientifiques des mathématiciens espagnols au 16e siecle?. Later on he returns to the same question when discussing Rey Pastor on Rezensionen, see [6], searching for a definite answer to the same question: "Welches sind die wissenschaftlichen Verdienste der spanischen Mathematiker des 16. Jahrhunderts?". Rey Pastor gives in [7] a powerful answer in the case of the algebraists.

Iberian mathematics. As Louis C. Karpinski points out[15], Mexico City and Lima emerged at the end of the 16th century as active centers of the Spanish culture, with a vital interest in literature, arts, sciences and technology[16].We have mentioned above the book "Physica speculatio" by Alonso de la Vera Cruz, published in México City in 1557, that introduced in the new world the calculatory systematization of Physics. About the same time, 1556, Juan Diez Freyle published in México City a manual of practical Arithmetic "Sumario compendioso de las cuentas de plata y oro que en los reynos del Pirú son necessarios a los mercaderes: y todo género de tratantes con algunas reglas tocantes al Arithmética". After that time, 1597, Joan de Belveder published in Lima a similar work "Libro general de las reducciones de plata y oro". A similar work appeared in México shortly after by Francisco Garreguilla.

The first treatise on Arithmetic that appeared in America was the "Arte para aprender todo el menor del Arithmetica sin maestro", by the accountant Pedro Paz, México City, 1623, see Florian Cajori [17]. Perhaps the most accomplished Mexican mathematician of the 17th century was the Latin American trained Don Carlos Sigüenza de Góngora, who held the chair of mathematics and astronomy at the university of México in 1672. Sigüenza in his treatise "Astronomica" reveals a remarkable familiarity with the European scientists and their writings. Among the most cited authors we find Kepler, Brahe, Hevelius, Descartes, Oldenburg and Huygens (see Karpinski).

A similar dimension was achieved by Don Pedro de Peralta Barnuevo in Lima during the early seventeen hundreds. Barnuevo held the chair of mathematics at the university of San Marcos. In astronomy he published a score of the astronomical calendars between 1721 and 1743 (Karpinski). Peralta Barnuevo was also an accomplished poet and dramatist[18] In the final analysis one may ask how did the works of the Hispanic American scholars compare with similar works published in the English colonies ? Let Louis Karpinski answer that question:

"Up to 1750 the Spanish American text books in Arithmetic compare favorably with those published in the English colonies. The majority of the early English arithmetics were reprints of English textbooks while the Spanish wrote independent textbooks introducing problems particularly adapted to the American situations. The English were more concerned to have the material adapted for instruction of children than to have them useful for adults. After 1750, the English work on mathematics forged rapidly ahead of the Spanish, both in quality and quantity. " (Karpinski op.cit. p.p. 62, 63.)

[15] "Mathematics in Latin America", Scripta Math., XIII, 1947, 59-63.

[16] Julio Rey Pastor in "La ciencia y la técnica en el descubrimiento de América", Madrid 1942, devotes a whole section of his book to metalurgy and mining as it was practiced in Hispanic America. What is more interesting, he includes a comparative study with the technologies used by other European countries.

[17] "The Earliest Arithmetic Published in America". Isis, 9, 391-401.

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