Revista de la
Unión Matemática Argentina
Volumen 36, 1990.

L^p-BOUNDEDNESS OF CERTAIN SINGULAR INTEGRAL OPERATORS

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INTRODUCTION. In [R-S], F.Ricci and E.Stein studied singular integral operators whose kernels are supported on $V \cup \{0\}$ where V is a connected analytic homogeneous submanifold of $\mathbf{R}^n$ not containing the origin.

It is also known that if B denotes the unit ball of $\mathbf{R}^m$, $g: B \to \mathbf{R}^n$ is a real analytic function, $g(0) = 0$, and k is a $C^\infty(\mathbf{R}^m - \{0\})$ function homogeneous of degree $-m$ and of mean value zero then the operator given by

$$Tf(x) = p.v.\int_B f(x-g(t))k(t)dt$$

is bounded on $L^p(\mathbf{R}^n)$, $1 < p < \infty$. See for example [C-N-S-W], [S-U].

Our aim now is to show that this result still holds even if g is not analytic at the origin but requiring it to be approximately homogeneous at that point in a sense that will be explicited at §2.

The proof follows the same lines that in the latter case with a more exhaustive use of lemma 2.1. stated in [M].

- Partially supported by CONICET and CONICOR.

It can be also proved with similar techniques that if g is approximately homogeneous in a neighborhood U of infinity then the operator given by

$$Tf(x) = \text{p.v.}\int_U f(x-g(t))k(t)dt$$

is bounded on $L^p(\mathbf{R}^n)$, $1 < p < \infty$. The case m=1 is proved in [S.UR.].

§ 2. We say that a real analytic function $g\colon B-\{0\} \to \mathbf{R}^n$ is approximately homogeneous at the origin if
$g(t) = (g_{a_1}(t)+\varphi_1(t),\dots,g_{a_n}(t)+\varphi_n(t))$ and it has the following properties

(2.1) If $1 \leqslant i \leqslant n$, $g_{a_i}(t)$ is a homogeneous function of degree a_i i.e. $g_{a_i}(rt) = r^{a_i} g_{a_i}(t)$, $t \in \mathbf{R}^m$, $r > 0$.

(2.2) If $m > 1$, the image of $g_0(t) = (g_{a_1}(t),\dots,g_{a_n}(t))$ generates $\mathbf{R}^n$ in the sense that it is not contained in any proper subspace of $\mathbf{R}^n$; if m=1, $g_0(\mathbf{R}^+)$ and $g_0(\mathbf{R}^-)$ generate $\mathbf{R}^n$.

(2.3) There exists $c > 0$ such that for each $t^0 \in \text{Dom } g$, φ_i has an analytic extension on $W_{t^0} = \{\xi \in \mathbf{C}^m / |\xi_i - t_i^0| < c \min_{t_j^0 \neq 0} |t_j^0|\}$.

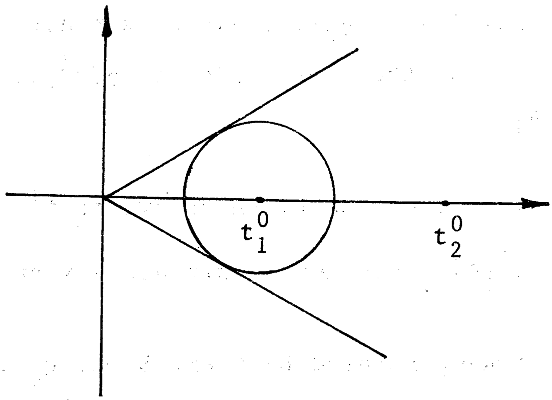
(2.4) For each $t^0 \in B-\{0\}$, $\lim_{r\to 0} r^{-a_i}\varphi_i(r\xi) = 0$ uniformly on W_{t^0}.

EXAMPLE. Let $f(t) = f_0(t) + \varphi(t)$ be the function defined on $\{t \in \mathbf{R} \;/\; 0 < |t| < 1\}$ by $f_0(t) = |t|^\lambda$, $\varphi(t) = |t|^{\lambda+\varepsilon}$ with $\lambda \geqslant 1$, $\varepsilon > 0$ and let $g(t_1,t_2) = (t_1,t_2,f(\sqrt{t_1^2+t_2^2}))$ be the revolution surface generated by the curve $(t,f(t))$.

We extend g to the region $W = \{(z_1,z_2)\,|\,\mathrm{Re}(z_1^2+z_2^2) > 0\}$ taking the principal argument to define z^λ.

Let us see that (2.3) and (2.4) hold.

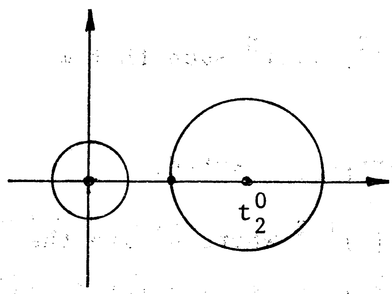
i) Let $t^0 = (t_1^0, t_2^0) \in B - \{0\}$. If $t_1^0 > 0$, $t_2^0 > 0$, $W_{t^0} \subset W$, with $c = \frac{1}{4}$. Indeed, $\{\xi_i \in \mathbf{C} / |\xi_i - t_i^0| \leqslant \frac{|t_i^0|}{2}\} \subset \{\xi_i \in \mathbf{C} / -\frac{\pi}{8} \leqslant \arg \xi_i \leqslant \frac{\pi}{8}\}$.



So for such ξ_i, $\mathbf{Re}\ \xi_i^2 > 0$ and then for $\xi = (\xi_1, \xi_2) \in W_{t^0}$, $\mathbf{Re}(\xi_1^2 + \xi_2^2) > 0$. If $t_1^0 > 0$, $t_2^0 < 0$, $\{\xi_2 / |\xi_2 - t_2^0| \leqslant \frac{|t_2^0|}{2}\} \subset$
$\subset \{\xi_2 \in \mathbf{C} / -\pi \leqslant \arg \xi_2 \leqslant -\frac{7}{8}\pi\} \cup \{\xi_2 \in \mathbf{C} / \frac{7}{8}\pi \leqslant \arg \xi_2 < \pi\}$.

Then $\frac{-\pi}{4} < \arg \xi_2^2 < \frac{\pi}{4}$.

We take now $t_1^0 = 0$ and $t_2^0 > 0$.



If $|\xi_1| \leqslant \frac{|t_2^0|}{4}$, $|\mathbf{Re}\ \xi_1^2| \leqslant$
$\leqslant |\xi_1^2| \leqslant \frac{|t_2^0|^2}{16}$.

If $|\xi_2 - t_2^0| \leqslant \frac{|t_2^0|}{2}$, $\mathbf{Re}\ \xi_2^2 =$
$= |\xi_2^2| \cos \arg \xi_2^2 \geqslant |\xi_2^2| \cos \frac{\pi}{4} \geqslant$
$\geqslant \frac{|t_2^0|^2}{4} \cdot \frac{\sqrt{2}}{2}$. Thus $\mathbf{Re}(\xi_1^2 + \xi_2^2) > |t_0^2|^2 \left(\frac{\sqrt{2}}{8} - \frac{1}{16}\right) > 0$.

ii) In this case $g_0(t_1, t_2) = (t_1, t_2, (t_1^2 + t_2^2)^{\lambda/2})$ and $a_1 = 1$,

$a_2 = 1, \; a_3 = \lambda$

$$r^{-\lambda}\,\varphi(r\sqrt{z_1^2+z_2^2}) = r^{-\lambda}\, r^{\lambda+\varepsilon}(z_1^2+z_2^2)^{\frac{\lambda+\varepsilon}{2}} \xrightarrow[r\to 0]{} 0 \quad \text{uniformly on } W_{t^0}.$$

NOTE 2.5. There exists $N \leqslant n$ such that the map given by $\phi_0(t^1,\dots,t^N) = g_0(t^1)+\dots+g_0(t^N)$ has rank n a.e. $(t^1,\dots,t^N) \in$ $\in B-\{0\}\times\dots\times B-\{0\}$ and the same holds for $\phi(t^1,\dots,t^N) =$ $= g(t^1)+\dots+g(t^N)$.

Proof. Since g_0 is not constant, there exists $t^1 \in B-\{0\}$ such that $\left.\frac{\partial g_0}{\partial t_1}\right|_{t^1},\dots,\left.\frac{\partial g_0}{\partial t_m}\right|_{t^1}$ are not simultaneously null in $\mathbf{R}^n$.

Let W be the subspace generated by them. Since g_0 is analytic on $B-\{0\}$ there exists a neighborhood U_{t^1} of t^1 where g_0 has a Taylor expansion. We state that there exists $t^2 \in U_{t^1}$ such that $\left.\frac{\partial g_0}{\partial t_i}\right|_{t^2} \notin W$ for some $1 \leqslant i \leqslant m$. Otherwise $D^\alpha g_0\big|_{t^2} \notin W$ for all multiindex α with $|\alpha| \geqslant 1$ and then $g_0(t) =$ $= g_0(t^1) + \sum_\alpha D^\alpha g_0\big|_{t^1} (t-t^1)^\alpha \quad \forall t \in U_{t^1}$ and so $g_0(t) \in g_0(t^1)+W$ $\forall\, t \in U_{t^1}$. This is a contradiction since g_0 is analytic on $B-\{0\}$ and generates $\mathbf{R}^n$.

Iterating this argument, we find $t^1,t^2,\dots,t^N$ such that ϕ_0 has rank n at $(t^1,\dots,t^N)$.

Now we define $\phi_k(t^1,\dots,t^N) = D_{2^k}\phi(2^{-k}t^1,\dots,2^{-k}t^N)$.

Let $J(t^1,\dots,t^N) = (\sum_i |A^i(t^1,\dots,t^N)|^2)^{1/2}$ where A^i are the minors of order n of the jacobian matrix of ϕ, and let J_k, A_k^i and J_0, A_0^i be the analogous functions for J_0 and ϕ_0 respectively; by 2.3) and 2.4) we observe that $J_k \xrightarrow[k\to\infty]{} J_0$ uniformly on $W_{t^1}\times\dots\times W_{t^N}$ and so $J(t^1,\dots,t^N) \neq 0$ a.e. $(t^1,\dots,t^N)$ i.e.

ϕ has rank n a.e. $(t^1,\ldots,t^N)$. □

Let k be a $C^\infty(\mathbf{R}^m-\{0\})$ function, homogeneous of degree -m and such that $\forall\ 0 < \varepsilon < \varepsilon' < 1 \quad \int_{\varepsilon<|t|<\varepsilon'} k(t)dt = 0.$

We define $Tf(x) = \text{p.v.} \int_B f(x-g(t))k(t)dt$ (2.6)

Following the same lines as in [S-U], we decompose T as in [R-S]. We take $\theta \in C_0^\infty(1/2,2)$ such that $\sum_{\ell=1}^\infty \theta(2^\ell s) = 1$. Then

$$k(t) = \sum_{\ell=1}^\infty k(t)\ \theta(2^\ell|t|) = \sum_{\ell=1}^\infty k_\ell(t) \quad , \quad t \in B.$$

Let μ_ℓ be the measure defined by

$$\mu_\ell(f) = \int_{|t|<1} f(g(t))\ k_\ell(t)dt.$$

We denote by $|\ |_n$ a homogeneous norm in $\mathbf{R}^n$ associated to the group of dilations given by $D_r(x_1,\ldots,x_n) = (r^{a_1}x_1,\ldots,r^{a_n}x_n)$ and we set $a = a_1+\ldots+a_n$. We also define $\theta_0(x) = \theta(|x|_n)$, $\theta_\ell(x) = 2^{\ell a}\ \theta_0(D_{2^\ell}x) = 2^{\ell a}\ \theta(2^\ell|x|_n)$. If $c = \int \theta(y)dy$ and $\eta_\ell(x) = c^{-1}(\theta_{\ell+1}(x)-\theta_\ell(x))$ then for each fixed j, we have that $\delta = \sum_{\ell=j}^\infty \eta_\ell + c^{-1}\theta_j$ in the sense of distributions and thus

$$Tf = \sum_{j=1}^\infty \mu_j * f = \sum_{j=1}^\infty \sum_{\ell=j}^\infty \eta_\ell * \mu_j * f + c^{-1}\sum_{j=1}^\infty \theta_j * \mu_j * f =$$

$$= \sum_{k=0}^\infty \sum_{j=1}^\infty \eta_{k+j} * \mu_j * f + c^{-1}\sum_{j=1}^\infty \theta_j * \mu_j * f = \sum_{k=0}^\infty M_k f + Lf.$$

NOTE 2.7. Set $\psi_\ell = \prod_{i=1}^N k_\ell(t^i)$. We observe that $\mu_\ell * \ldots * \mu_\ell$ N times is the measure given by $\mu_\ell*\ldots*\mu_\ell(f) =$ $= \int f(\phi(t^1,\ldots,t^N)).\psi_\ell(t^1,\ldots,t^N)dt^1\ldots dt^N$ i.e. it is the transported measure of ψ_ℓ by ϕ.

ϕ has rank n a.e. and it is analytic on supp ψ_ℓ then $\mu_\ell * \ldots * \mu_\ell$ N times is absolutely continuous with a density ρ_ℓ satisfying

$$\int_{R^n} |\rho_\ell(x+y)-\rho_\ell(x)|dx < c(\int|\nabla\psi_\ell| + \int|\psi_\ell|)^\sigma[(\int|\psi_\ell(t)J_\ell dt|)^{\frac{2\sigma}{1-\sigma}} dt]^{1-\sigma} \quad (2.8)$$

for some $\sigma > 0$, c depending only on ϕ. For a proof see [R-S] and [S].

THEOREM 2.9. *Let* k *be a* $C^\infty(\mathbf{R}^m-\{0\})$ *function homogeneous of degree* -m *such that if* $0 < \varepsilon < \varepsilon' < 1$ $\int_{\varepsilon<|t|<\varepsilon'} k(t)dt = 0$.
Let $g(t) = (g_{a_1}(t)+\varphi_1(t),\ldots,g_{a_n}(t)+\varphi_n(t))$ *satisfying* 2.1) *to* 2.4). *Then the operator* T *given by* (2.6) *is bounded on* $L^p(\mathbf{R}^n)$, $1 < p < \infty$.

Proof. We now follow straightforward the proof of theorem 3.2 in [S.U]. So we decompose $T = \sum_{k\geq 0} M_k + L$ and we have that

$$\forall\ 0 < \varepsilon < 1 \qquad \|M_k\|_{p,p} \leqslant c_{\varepsilon,p} 2^{\varepsilon k} \quad \text{and} \quad \|L\|_{p,p} \leqslant c.$$

The following step is to prove that:

There exists $\sigma > 0$ such that $\|M_k\|_{2,2} \leqslant c\ 2^{-\sigma k}$. (2.10)

Then the theorem will follow by interpolation and duality arguments.

Using Cotlar's lemma and the iteration argument in [Ch], it is enough to prove that for $\ell < j$

$$\|\rho_\ell * \mu_j^* * \eta_{k+j}^*\|_1 \leqslant c\ 2^{-\sigma k}\ 2^{(\ell-j)\sigma} \quad \text{for some } \sigma > 0.$$

We define $\tilde{\psi}(t^1,\ldots,t^N) = 2^{-Nm}\ \psi_\ell(2^{-\ell}t^1,\ldots,2^{-\ell}t^N)$ so that

$\tilde{\rho}_\ell(y) = 2^{-\ell a}\ \rho_\ell(2^{-\ell}y)$ is the density of the transported measure of $\tilde{\psi}$ by ϕ_ℓ and supp $\tilde{\psi} \subset \{(t^1,\ldots,t^N)/\ \frac{1}{2} < |t^i| < 2\}$.

If we prove that

$$\int_{R^n} |\tilde{\rho}_\ell(x+y)-\tilde{\rho}_\ell(x)|dx < c \ |y|_n^\sigma \quad \text{for some} \quad c,\sigma > 0$$

independent of ℓ, the rest of the proof of (2.10) follows as in [S.U].

By (2.9) it is enough to show that

$$\int_{\mathrm{supp}\ \tilde{\psi}} |J_\ell(t)|^{-\alpha} dt < c \tag{2.11}$$

independent of ℓ, for some $\alpha > 0$.

As $J_\ell(t) \neq 0$ a.e. $t \in \mathrm{supp}\ \tilde{\psi}$, there exists a minor, that we denote by A_ℓ, such that $A_\ell \neq 0$ a.e. $t \in \mathrm{supp}\ \tilde{\psi}$.

To obtain (2.11) it is enough to check that $\int_{\mathrm{supp}\,\tilde{\psi}} |A_\ell(t)|^{-\alpha} dt < C$ for some $\alpha > 0$, $\forall\ \ell$ large enough.

Since

$$D\phi(t^1,\dots,t^N) = \begin{pmatrix} \left(\frac{\partial g_{a_1}}{\partial t_1} + \frac{\partial \varphi_1}{\partial t_1}\right)\Big|_{t^1} & \cdots\cdots & \left(\frac{\partial g_{a_1}}{\partial t_m} + \frac{\partial \varphi_1}{\partial t_m}\right)\Big|_{t^N} \\ \vdots & & \vdots \\ \left(\frac{\partial g_{a_n}}{\partial t_i} + \frac{\partial \varphi_n}{\partial t_1}\right)\Big|_{t^1} & \cdots\cdots & \left(\frac{\partial g_{a_n}}{\partial t_m} + \frac{\partial \varphi_n}{\partial t_m}\right)\Big|_{t^N} \end{pmatrix}$$

it is easy to see, by induction on n, that $A(t^1,\dots,t^N) = A_0(t^1,\dots,t^N) + R(t^1,\dots,t^N)$ where A_0 is homogeneous of degree $(a-n)$ and $R(t^1,\dots,t^N)$ is a finite sum of terms of the form

$$\frac{\partial g_{a_{i_1}}}{\partial t_{\ell_1}}\Big|_{t^{k_1}} \cdots \frac{\partial g_{a_{i_s}}}{\partial t_{\ell_s}}\Big|_{t^{k_s}} \cdots \frac{\partial \varphi_{j_1}}{\partial t_{\ell_{s+1}}}\Big|_{t^{k_{s+1}}} \cdots \frac{\partial \varphi_{j_{(n-s)}}}{\partial t_{\ell_n}}\Big|_{t^{k_n}} \tag{2.12}$$

with $1 \leqslant i_1,\dots,i_s,j_1,\dots,j_{n-s} \leqslant n$, $\{i_1,\dots,i_s\} \cap \{j_1,\dots,j_{n-s}\} = \phi$, $s < n$, $1 \leqslant \ell_1,\dots,\ell_n \leqslant m$, $1 \leqslant k_1,\dots,k_n \leqslant N$. Moreover, 2.12 is a function of separated variables $t^1,\dots,t^N$ and each variable appears at most in m factors. Thus, if $r = 2^{\ell}$, we have that $A_r(t^1,\dots,t^N) = r^{-(a-n)}A(r\,t) = A_0(t^1,\dots,t^N) + r^{-(a-n)}R(rt^1,\dots,rt^N)$.

We want to prove that for each $t_0 = (t_0^1,\dots,t_0^N) \in \mathrm{supp}\tilde{\psi}$ there exists a neighborhood of t_0, U_{t_0}, and constants α_0, c_0 such that

$$\int_{U_{t_0}} |A_0(t) + r^{-(a-n)}R(rt)|^{-\alpha_0}\,dt \leq c_0 \quad \forall\ r \leq r_0. \qquad (2.13)$$

Provided (2.13) the theorem follows since $\mathrm{supp}\tilde{\psi}$ is compact. To verify (2.13) we will make use of Lemma 2.1 in [M]. Indeed, we will check that given t_0 and $\delta > 0$ there exists $r_0 > 0$ such that

$$\sum_J r^{-(a-n)} \frac{r^{|J|}}{J!} \left| \frac{D^J R}{dt^J}(r\,t_0) \right| M^{|J|} \leq \delta \qquad (2.14)$$

for a suitable choice of M, $\forall\ r \leq r_0$.

To obtain (2.14) we analize only one summand of $R(t^1,\dots,t^N)$. By (2.12) this term has the form $f_1(t)\dots f_n(t)$ where for $1 \leq k \leq s$ f_k is a first partial derivate of $g_{a_{i_k}}$ and for $s < k \leq n$, f_k is a first partial derivate of $\varphi_{j_{(k-s)}}$.

If $J = (j_1,\dots,j_{mN})$, $D^J(f_1\dots f_n) =$

$$= \sum_{|I_1| = j_1,\dots,|I_{mN}| = j_{mN}} \frac{J!}{I_1!\dots I_{mN}!} D^{i_1^{mN}\dots i_1^1} f_1 \dots D^{i_n^{mN}\dots i_n^1} f_n$$

where $I_k = (i_1^k,\dots,i_n^k)$ for $1 \leq k \leq mN$. Let $N_\ell = (i_\ell^1,\dots,i_\ell^{mN})$ for $1 \leq \ell \leq n$. Then $|N_1| +\dots+ |N_n| = |J|$.

We also write $r^{-(a-n)}\, r^{|J|} = r^{-a_{i_1}}\, r^{|N_1|+1} \dots r^{-a_{i_s}}\, r^{|N_s|+1}$.

$\cdot\ r^{-a_{j_1}}\, r^{|N_{s+1}|+1} \dots r^{-a_{j_{(n-s)}}}\, r^{|N_n|+1}$.

We must estimate $D^{N_i} f_i$. For example we take $i = n$ and we assume that $j_{(n-s)} = n$. So $D^{N_n} f_n = D^{M_n} \varphi_n$ where $|M_n| = |N_n|+1$ and we must evaluate $|D^{M_n} \varphi_n(rx^0)|$ for $x^0 \in B-\{0\}$.

We apply Cauchy formula.

Let D_{x_0} denote the polydisk $\{\xi \mid |\xi_i - x_i^0| = c \min_{x_j^0 \neq 0} |x_j^o|\}$

$$D^{M_n}\varphi_n(rx^0) = \frac{M_n!}{(2\pi i)^m} \int_{D_{rx^0}} \frac{\varphi_n(\xi)}{(\xi - rx^0)^{M_n+(1,\dots,1)}} d\xi$$

where c is as in (2.3).

$$r^{-a_n} r^{|N_n|+1} |D^{M_n}\varphi_n(rx^0)| \leqslant M_n!\, r^{-a_n} \sup_{D_{rx^0}} |\varphi_n(\xi)|\, c^{-|M_n|} . (\min_{x_j^0 \neq 0} |x_j^0|)^{-M_n}.$$

By the hypothesis (2.4) about φ_n we have that

$$r^{-a_n} \sup_{D_{rx^0}} |\varphi_n(\xi)| = r^{-a_n} \sup_{D_{rx^0}} |\varphi_n(r\, r^{-1}\xi)| \leqslant \varepsilon \quad \forall\ r \text{ small}$$

enough since $r^{-1}\xi$ belong to W_{x^0}.

We obtain similar estimations for the others f_k, but if $1 \leqslant k \leqslant s$ instead of $j_{(k-s)}$ we have $g_{a_{i_k}}$ which is a homogeneous function of degree a_{i_k} and thus $r^{-a_{i_k}} \sup|g_{a_{i_k}}| \leqslant C$.

Returning to (2.14) we have that the sum is bounded by

$$C\, \varepsilon^{n-s} \sum_J \sum_{|J_1| = j_1, \dots, |J_{mN}| = j_{mN}} |J|^n c^{-|J|} M^{|J|} (\min_{x_j^0 \neq 0} |x_j^0|)^{-|J|}$$

$$< C\, \varepsilon^{n-s} \sum_J \frac{1}{2^{|J|}} |J|^{n+mnN} \quad \forall\ r \text{ small enough choosing}$$

$$M < \frac{1}{2} c \min_{x_j^0 \neq 0} |x_j^0|.$$

Since the last sum is convergent the theorem follows.

Now we state an analogous result in the case that g is approximately homogeneous in a neighborhood U of infinity. More precisely, let $g: U \to \mathbf{R}^n$ be a real analytic function of the form $g(t) = (g_{a_1}(t) + \varphi_1(t),\ldots,g_{a_n}(t) + \varphi_n(t))$ satisfying (2.1), (2.2), (2.3) and

(2.15) for each $t_0 \in U$, $\lim_{r\to+\infty} r^{-a_i}\varphi_i(r\xi) = 0$ uniformly on W_{t_0},

instead of (2.4).

THEOREM 2.16. *The operator* T *defined by* $Tf(x) = p.v \int_U f(x-g(t))k(t)dt$ *is bounded on* $L^p(\mathbf{R}^n)$, $1 < p < \infty$.

Sketch of the proof. We decompose $T = \sum_{k>0} M_k + L$ and we obtain that $\|L\|_{p,p} < C$, $\|M_k\|_{p,p} < C\,2^{\varepsilon k}$ $\forall\ \varepsilon > 0$, $1 < p < \infty$, as in [S.UR.]. The proof that $\|M_k\|_{2,2} < C\ 2^{-\sigma k}$ for some $\sigma > 0$ is completely analogous to the given in theorem 2.9.

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Recibido en julio de 1990.