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SOLVABILITY AND UNIQUENESS RESULTS FOR EQUATIONS OF MEAN CURVATURE TYPE

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ABSTRACT

We study the mean curvature equation $\Delta X = 2HX_u \wedge X_v$. For any fixed $H = H(u, v, X, X_u, X_v)$ we give a family of boundary data g such that the Dirichlet problem is solvable. Furthermore, we prove under some conditions local and global uniqueness of the solutions in the Banach Space $C^1(\overline{\Omega}, \mathbb{R}^3)$.

1. INTRODUCTION

We consider the Dirichlet problem in a bounded $C^{1,1}$ domain $\Omega \subset \mathbb{R}^2$ for a vector function $X : \overline{\Omega} \longrightarrow \mathbb{R}^3$ which satisfies the equation of prescribed mean curvature

$$(1) \begin{cases} \Delta X = 2H(u, v, X, X_u, X_v)X_u \wedge X_v & \text{in} \quad \Omega \\ X = g & \text{in} \quad \partial\Omega \end{cases}$$

where X_u and X_v are the partial derivatives of X, $\wedge$ denotes the exterior product in $\mathbb{R}^3$, $H : \overline{\Omega} \times (\mathbb{R}^3)^3 \longrightarrow \mathbb{R}$ is a given continuous function and the boundary data g belongs to $W^{2,p}(\Omega, \mathbb{R}^3)$ for some $2 < p < \infty$. As the usual prescription of the Dirichlet data, we may assume without loss of generality that g is harmonic.

Problem (1) arises in the Plateau and Dirichlet problems for the prescribed mean curvature equation that has been studied by variational methods for constant H in [BC], [H], [S] and for $H = H(X)$ in [ADNM], [W]. Indeed, it is possible to find solutions of (1) by minimizing the functional $D_H(X) = D(X) + 2V(X)$, where D is the Dirichlet integral and V is the Hildebrandt's volume given by $V(X) = \frac{1}{3}\int_B Q(X)X_u \wedge X_v$, with $divQ = 3H$. However, for the general case a functional for (1) is not known. Solutions by topological methods are given in [AM], and in [ABMR] for a nonparametric surface. For H and g constant, the solution $X \equiv g$ is unique [We]. A nonuniqueness result for constant $H > 0$ and nonconstant g with $H\|g\|_\infty < 1$ is given in [BC].

In order to study problem (1) we consider the operator $T : C^1(\overline{\Omega}, \mathbb{R}^3) \longrightarrow C^1(\overline{\Omega}, \mathbb{R}^3)$ given by $T\overline{X} = X$, where $X \in W^{2,p} \hookrightarrow C^1$ is the unique solution of the linear problem

$$\begin{cases} \Delta X = 2H(u, v, \overline{X}, \overline{X}_u, \overline{X}_v)\overline{X}_u \wedge \overline{X}_v & \text{in} \quad \Omega \\ X = g \quad \text{in} \quad \partial\Omega \end{cases}$$

Thus, any strong solution of (1) may be regarded as a fixed point of T. We remark that any weak solution in $W^{1,\infty}(\Omega, \mathbb{R}^3)$ is strong, and classical if g and $\partial\Omega$ are smooth (see [AM]).

2. SOLVABILITY OF (1)

In order to find a fixed point of T, we recall the apriori bound c_Δ for the Laplacian, i.e. $\|X\|_{2,p} \le c_\Delta\|\Delta X\|_p$ for every $X \in W^{2,p} \cap W_0^{1,p}$ (see [GT]), and remark that by the compactness of the imbedding $W^{2,p} \hookrightarrow C^1$ the operator T is compact. Thus, by Schauder's Theorem it suffices to find R such that $T(B_R(g)) \subset B_R(g)$. For the sake of simplicity, for $X \in C^1$ we define $H|_X : \overline{\Omega} \to \mathbb{R}$ given by $H|_X(u,v) = H(u, v, X, X_u, X_v)$, and we say that H is Lipschitz on $X \in C^1(\overline{\Omega}, \mathbb{R}^3)$ if there exist $R > 0$ and $k = k(R)$ such that

$$\|H|_Y - H|_X\|_p \le k\|Y - X\|_{1,\infty} \qquad \text{for any } Y \in B_R(X).$$

THEOREM 1

Let K be a compact subset of $\mathbb{R}^3$. Then there exists a constant $c_\infty = c_\infty(H, K)$ such that (1) is solvable for any $g \in A_{K,c_\infty}$, with

$$A_{K,c_\infty} = \{g \in W^{2,p}(\Omega, \mathbb{R}^3) : g(\overline{\Omega}) \subset K, \|\nabla g\|_\infty \le c_\infty\}$$

<u>*Proof*</u>

For $R > 0$, and fixed $\overline{c} \in \mathbb{R}$ we define $H_R = \sup_{g \in A_{K,\overline{c}}} h_R(g) \in \mathbb{R}$, where $h_R(g)$ is given by

$$h_R(g) = \sup_{\|X - g\|_{1,\infty} \le R} \|H|_{\overline{X}}\|_p$$

As H_R is nondecreasing we may fix $\overline{R}$ such that $\overline{R}H_{\overline{R}} = \frac{1}{4c_1c_\Delta}$, where c_1 is the constant of the imbedding $W^{2,p} \hookrightarrow C^1$. Thus, for $g \in A_{K,\overline{c}}$, if $\|\overline{X} - g\|_{1,\infty} \leq R \leq \overline{R}$ we have:

$$\|T\overline{X} - g\|_{1,\infty} \leq c_1c_\Delta \|H|_{\overline{X}}\|_p \|\nabla \overline{X}\|_\infty^2 \leq c_1c_\Delta H_R(R + \|\nabla g\|_\infty)^2$$

As $H_R \leq H_{\overline{R}}$, it suffices to find $R \in (0, \overline{R}]$ such that $c_1c_\Delta H_{\overline{R}}(R + \|\nabla g\|_\infty)^2 \leq R$. Setting $c_\infty = \min\{\overline{c}, \frac{1}{4c_1c_\Delta H_{\overline{R}}}\}$, it's immediate to see that the parabola

$$R^2 + (2\|\nabla g\|_\infty - \frac{1}{c_1cH_{\overline{R}}})R + \|\nabla g\|_\infty^2$$

admits the positive root

$$R_0 = \frac{1}{2c_1cH_{\overline{R}}}\left(1 - \sqrt{1 - 4c_1c_\Delta H_{\overline{R}}\|\nabla g\|_\infty} - 2c_1cH_{\overline{R}}\|\nabla g\|_\infty\right)$$

Let $x = \|\nabla g\|_\infty$ and consider the function

$$F(x) = \frac{1}{2c_1cH_{\overline{R}}}\left(1 - \sqrt{1 - 4c_1cH_{\overline{R}}x} - 2c_1cH_{\overline{R}}x\right)$$

Then, as $F'(x) \geq 0$ for $0 \leq x < \frac{1}{4c_1cH_{\overline{R}}}$, we conclude that $R_0 \leq F(\frac{1}{4c_1cH_{\overline{R}}}) = \frac{1}{4c_1cH_{\overline{R}}} = \overline{R}$, and the proof is complete. □

REMARKS:

i) As the proof of Theorem 1 does not depend on the choice of $\overline{c}$, *a sharper value for* c_∞ *may be obtained taking* $\max_{\overline{c}\in\mathbb{R}}\left(\min\{\overline{c}, \frac{1}{4c_1c_\Delta H_{\overline{R}}(\overline{c})}\}\right)$.

ii) We may also observe that

$$\|T\overline{X} - g\|_{1,\infty} \leq 2c_1c_\Delta \|H|_{\overline{X}}\|_\infty \|\overline{X}_u \wedge \overline{X}_v\|_p \leq c_1c_\Delta \|H|_{\overline{X}}\|_\infty \|\nabla \overline{X}\|_{2p}^2$$

Then, if we define

$$h_R^\infty(g) = \sup_{\|X-g\|_{1,\infty}\leq R} \|H|_{\overline{X}}\|_\infty, \qquad H_R^\infty = \sup_{g(\overline{\Omega})\subset K, \|\nabla g\|_{2p}\leq \overline{c}} h_R^\infty(g)$$

it follows that

$$\|T\overline{X} - g\|_{1,\infty} \leq c_1c_\Delta H_R^\infty(\|\nabla g\|_{2p} + |\Omega|^{1/2p}R)^2$$

Thus, taking $\overline{R}(\overline{c})$ *such that* $\overline{R}H^{\infty}_{\overline{R}} = \dfrac{1}{4|\Omega|^{1/2p}c_1c_\Delta}$, *(1) is solvable for any* g *belonging to the set* $\{g \in W^{2,p}(\Omega, \mathbb{R}^3) : g(\overline{\Omega}) \subset K, \|\nabla g\|_{2p} \leq c_{2p}\}$, *with*

$$c_{2p} = \sup_{\overline{c}\in\mathbb{R}} \left(\min\{\overline{c}, \frac{1}{4|\Omega|^{1/2p}c_1c_\Delta H^{\infty}_{\overline{R}}(\overline{c})}\} \right)$$

iii) If H *is bounded with respect to* X, *then* c_∞ *and* c_{2p} *may be choosen independently of* K.

iv) In some cases it holds that $c_\infty = c_{2p} = +\infty$. *For example, for* $H = \frac{H_1(u,v,X)}{1+\nabla X^2}$, *with* $|H_1(u,v,x)| \leq r|x| + s$ *and* $r < \frac{1}{c_1 c}$, *it is clear that if* $X = \sigma TX$ *for some* $0 \leq \sigma \leq 1$, *then*

$$\|X - \sigma g\|_{1,\infty} \leq 2c_1c_\Delta\sigma\|\frac{H_1(u,v,X)}{1+\nabla X^2}X_u \wedge X_v\|_p \leq c_1c_\Delta(r\|X\|_{1,\infty} + s)$$

This proves that $\|X\|_{1,\infty} \leq M$ *for some constant* M, *and the result follows by Leray-Schauder's Theorem.*

3. LOCAL UNIQUENESS OF THE SOLUTIONS

THEOREM 2

Let $\overline{Y} \in C^1(\overline{\Omega}, \mathbb{R}^3)$, *and assume that* H *is Lipschitz on* $\overline{Y}$ *with constant* k *and radius* R. *We define*

$$c(\overline{Y}) = 2h_R(\overline{Y})(2\|\nabla\overline{Y}\|_\infty + R) + k\|\nabla\overline{Y}\|^2_\infty$$

Then, for $\overline{X} \in B_R(\overline{Y})$:

$$\|T(\overline{X}) - T(\overline{Y})\|_{1,\infty} \leq c_1c_\Delta c(\overline{Y})\|\overline{X} - \overline{Y}\|_{1,\infty}$$

In particular, if $\overline{Y}$ *is a solution of (1) and* $c_1c_\Delta c(\overline{Y}) < 1$, *then* $\overline{Y}$ *is unique in* $B_R(\overline{Y})$.

Proof

Given $\overline{X} \in B_R(\overline{Y})$, it follows that

$$\|T(\overline{X}) - T(\overline{Y})\|_{1,\infty} \leq 2c_1c\|H|_{\overline{X}}\overline{X}_u \wedge \overline{X}_v - H|_{\overline{Y}}\overline{Y}_u \wedge \overline{Y}_v\|_p$$

$$\leq 2c_1c(\|H|_{\overline{X}}\left((\overline{X}_u - \overline{Y}_u) \wedge \overline{X}_v + \overline{Y}_u \wedge (\overline{X}_v - \overline{Y}_v)\right)\|_p + \|(H|_{\overline{X}} - H|_{\overline{Y}})\overline{Y}_u \wedge \overline{Y}_v\|_p)$$

As $\|\nabla(\overline{X}-\overline{Y})\|_\infty \leq R$, the result follows.

Furthermore, if $c_1 c_\Delta c(\overline{Y}) < 1$, and $\overline{X} \neq \overline{Y} \in B_R(\overline{Y})$ are fixed points of T then $\|\overline{X}-\overline{Y}\|_{1,\infty} < \|\overline{X}-\overline{Y}\|_{1,\infty}$, a contradiction. □

REMARK:

In particular, for $H = H(u,v)$, $c(\overline{Y}) = 2\|H\|_p(2\|\nabla\overline{Y}\|_\infty + R)$ *for any* R. *Hence, any solution* $\overline{Y}$ *such that* $\|\nabla\overline{Y}\|_\infty < \frac{1}{2\|H\|_p c_1 c}$ *is isolated.*

The following result shows that if H is Lipschitz near a constant, the existence condition given in Theorem 1 may be stated in more precise terms. Moreover, if $\|g-a\|_{1,\infty}$ is small, then T is a contraction in $B_R(g)$:

COROLLARY 3

Let $a \in \mathbb{R}^3$, *and assume that* H *is Lipschitz on* $B_{R_0}(a)$ *with constant* k *and radius* R_0. *Let* $H_a = \|H|_a\|_p = \|H(u,v,a,0,0)\|_p$. *Then:*

a) (1) admits a solution in $B_R(g)$ *for any harmonic* $g \in W^{2,p}(\Omega,\mathbb{R}^3)$ *such that* $\|g-a\|_{1,\infty} = R \leq \min\{\frac{R_0}{2}, \frac{\sqrt{H_a^2+\frac{2k}{c_1 c_\Delta}}-H_a}{4k}\}$.

b) (1) admits a unique solution in $B_R(g)$ *for any harmonic* $g \in W^{2,p}(\Omega,\mathbb{R}^3)$ *such that* $\|g-a\|_{1,\infty} = R \leq \min\{\frac{R_0}{2}, \frac{\sqrt{2H_a^2+\frac{(1+2\sqrt{2})k}{c_1 c_\Delta}}-\sqrt{2}H_a}{2k(1+2\sqrt{2})}\}$.

Proof

If $\|g-a\|_{1,\infty} = R \leq \frac{R_0}{2}$, then $B_R(g) \subset B_{R_0}(a)$ and as in Theorem 2 we obtain, for $\overline{X} \in B_R(g)$:

$$\|T(\overline{X})-g\|_{1,\infty} \leq 2c_1 c_\Delta \|H|_{\overline{X}} \overline{X}_u \wedge \overline{X}_v\|_p \leq 4c_1 c_\Delta h_R(g) R^2$$

Then $T(B_R(g)) \subset B_R(g)$ for $h_R(g)R \leq \frac{1}{4c_1 c_\Delta}$ and as $h_R(g) \leq h_{2R}(a) \leq H_a + 2kR$ part a) follows. Furthermore, if also $\overline{Y} \in B_R(g)$ we have:

$$\|T(\overline{X})-T(\overline{Y})\|_{1,\infty} \leq c_1 c_\Delta (2\|H|_{\overline{X}}\|_p \|\nabla(\overline{X}-\overline{Y})\|_\infty \|(\overline{X}_v^2+\overline{Y}_u^2)^{1/2}\|_\infty$$

$$+k\|\nabla\overline{Y}\|_\infty^2 \|\overline{X}-\overline{Y}\|_{1,\infty}) \leq 4c_1 c_\Delta (\sqrt{2}h_R(g)R + kR^2)\|\overline{X}-\overline{Y}\|_{1,\infty}$$

and then T is a contraction in $B_R(g)$ for $\sqrt{2}h_R(g)R + kR^2 < \frac{1}{4c_1 c}$. Thus, b) is proved. □

In the next theorem we obtain the local uniqueness under different assumptions. For this purpose, we define for $X_0, X \in C^1(\overline{\Omega},\mathbb{R}^3)$ and $Z(u,v) = (Z_1, Z_2, Z_3) \in (\mathbb{R}^3)^3$ the matrices given by

$$AW = 2\left(H|_X W \wedge X_{0_v} + (\frac{\partial H}{\partial X_u}(u, v, Z)W)X_{0_u} \wedge X_{0_v}\right)$$

$$BW = 2\left(H|_X X_{0_u} \wedge W + (\frac{\partial H}{\partial X_v}(u, v, Z)W)X_{0_u} \wedge X_{0_v}\right)$$

$$CW = 2(\frac{\partial H}{\partial X}\Big|_Z .W)X_{0_u} \wedge X_{0_v}$$

and the number

$$r := \left(\frac{\||A|^2 + |B|^2\|_\infty}{\lambda_1}\right)^{1/2}$$

where λ_1 is the first eigenvalue of $-\Delta$.

In particular, for $X = X_0$, $Z = (X_0, X_{0u}, X_{0v})$ we will write $A(X_0)$, $B(X_0)$, $C(X_0)$ and $r(X_0)$ respectively.

THEOREM 4

Let $X_0 \in C^1(\overline{\Omega}, \mathbb{R}^3)$ be a solution of (1) such that one of the following conditions holds:

i) H is Lipschitz on X_0 with constant $\dfrac{\delta}{2c_1c_0\|X_{0_u} \wedge X_{0_v}\|_\infty}$ for some $\delta < 1$, some constant c_0 depending on X_0 and

$$\|H|_{X_0} X_{0_u}\|_\infty + \|H|_{X_0} X_{0_v}\|_\infty < \frac{\sqrt{\lambda_1}}{2}$$

ii) H is continuously differentiable on X_0 with respect to X, X_u and X_v, and $r(X_0) < 1$, $C(X_0) \geq -\kappa > \lambda_1(r-1)$.

Then X_0 is isolated in $C^1(\overline{\Omega}, \mathbb{R}^3)$.

<u>*Proof*</u>

If X is another solution of (1), then $Y = X - X_0$ is a solution of the equation

$$\begin{cases} \Delta Y = 2H|_{Y+X_0}(Y + X_0)_u \wedge (Y + X_0)_v - 2H|_{X_0} X_{0_u} \wedge X_{0_v} & \text{in} \quad \Omega \\ Y = 0 \quad \text{in} \quad \partial\Omega \end{cases}$$

Let S be the operator given by $S(Y) = 2H|_{X_0}(X_{0_u} \wedge Y_v + Y_u \wedge X_{0_v})$, then:

$$LY := \Delta Y - S(Y) = 2H|_{Y+X_0} Y_u \wedge Y_v + 2(H|_{Y+X_0} - H|_{X_0})(X_{0_u} \wedge Y_v + Y_u \wedge X_{0_v})$$

$$+2\left(H|_{Y+X_0} - H|_{X_0}\right) X_{0_u} \wedge X_{0_v}$$

If i) holds, L satisfies the conditions of lemma 6 below, and then we obtain, for small $0 < R = \|Y\|_{1,\infty}$:

$$R \le c_1 c_0 \|LY\|_p \le kR^2 + \delta R$$

for a constant k. Leting $R \longrightarrow 0$ we obtain $1 \le \delta$, a contradiction. This shows that X_0 is isolated.

On the other hand, if ii) holds we have:

$$H|_{Y+X_0} - H|_{X_0} = \frac{\partial H}{\partial X}\Big|_{X_0} Y + \frac{\partial H}{\partial X_u}\Big|_{X_0} Y_u + \frac{\partial H}{\partial X_v}\Big|_{X_0} Y_v + \psi(Y)$$

and the result follows in a similar way, from the equation

$$\overline{L}Y = 2H|_{Y+X_0} Y_u \wedge Y_v + 2\psi(Y) X_{0_u} \wedge Y_v + Y_u \wedge X_{0_v} + 2\left(H|_{Y+X_0} - H|_{X_0}\right) X_{0_u} \wedge X_{0_v}$$

with $\overline{L}Y = \Delta Y - A(X_0)Y_u - B(X_0)Y_v - C(X_0)Y$. □

REMARK:

In the previous theorem, the condition $C \ge 0$ holds if and only if $X_{0_u} \wedge X_{0_v}$ is orthogonal to $\frac{\partial H}{\partial X}\Big|_{X_0}$. Indeed, writing $\frac{\partial H}{\partial X}\Big|_{X_0} = (h_1, h_2, h_3)$, $X_{0_u} \wedge X_{0_v} = (x_1, x_2, x_3)$, we see that $0 \le Ce_i.e_i = 2h_i x_i$, and for $Y = (t, 1, 0)$ we obtain that $(th_1 + h_2)(tx_1 + x_2) \ge 0$ for every t. Then $0 \ge (h_1 x_2 + h_2 x_1)^2 - 4h_1 h_2 x_1 x_2 = (h_1 x_2 - h_2 x_1)^2$. This implies that $h_1 x_2 = h_2 x_1$, and in the same way we prove that $h_2 x_3 = h_3 x_2$, $h_1 x_3 = h_3 x_1$. The converse is immediate.

4. GLOBAL UNIQUENESS

THEOREM 5

Let $X_0 \in C^1(\overline{\Omega}, \mathbb{R}^3)$ be a solution of (1) such that H is continuously differentiable with respect to X, X_u and X_v and assume that $r < 1$ and $C \ge -\kappa > \lambda_1(r-1)$ for any $X \in C^1(\overline{\Omega}, \mathbb{R}^3)$, $Z \in (\mathbb{R}^3)^3$. Then X_0 is unique in $C^1(\overline{\Omega}, \mathbb{R}^3)$.

Proof

We proceed as in theorem 4: if $X_0, X \in C^1(\overline{\Omega}, \mathbb{R}^3)$ are solutions of (1), then for $Y = X - X_0$ we have:

$$\Delta Y - 2H|_X \left(Y_u \wedge X_{0_v} + X_u \wedge Y_v\right) - 2(H|_X - H|_{X_0}) X_{0_u} \wedge X_{0_v} = 0$$

Moreover, by the mean value theorem we have, for fixed (u,v):

$$H|_X - H|_{X_0} = \frac{\partial H}{\partial X}(u,v,Z)Y + \frac{\partial H}{\partial X_u}(u,v,Z)Y_u + \frac{\partial H}{\partial X_v}(u,v,Z)Y_v$$

for some $Z = Z(u,v) \in L^\infty(\Omega,(\mathbb{R}^3)^3)$, and then $LY := \Delta Y - AY_u - BY_v - CY = 0$. As $Y = 0$ in $\partial\Omega$, we conclude from lemma 6 that $Y \equiv 0$. □

REMARK

For example, conditions of theorem 5 hold for $H = \frac{H_1(u,v)}{1+|\nabla X|^2}$, *with* H_1 *small enough. From a previous remark, also existence holds in this case. In particular, for constant* g *existence and uniqueness can be proved when* $\|H_1\|_2 > \frac{\sqrt{\lambda_1}}{2c_0}$.

5. A TECHNICAL LEMMA

In this section we extend a well-known result for linear elliptic second order operators in $W^{2,p}(\Omega,\mathbb{R})$:

LEMMA 6

Let $L : W^{2,p}(\Omega,\mathbb{R}^3) \longrightarrow L^p(\Omega,\mathbb{R}^3)$ *be the linear elliptic operator given by* $LX = \Delta X + AX_u + BX_v + CX$, *with* $A,B,C \in L^\infty(\Omega,\mathbb{R}^{3\times 3})$, $2 < p < \infty$ *and assume that* $r := \left(\frac{\||A|^2+|B|^2\|_\infty}{\lambda_1}\right)^{1/2} < 1$ *and* $C \le \kappa < \lambda_1(1-r)$, *where* λ_1 *is the first eigenvalue of* $-\Delta$.

Then there exists a constant c *such that*

$$\|X\|_{2,p} \le c\|LX\|_p$$

for every $X \in W^{2,p} \cap W_0^{1,p}(\Omega,\mathbb{R}^3)$.

<u>*Proof*</u>

Let $Z_n \in W^{2,p} \cap W_0^{1,p}(\Omega,\mathbb{R}^3)$ be a sequence such that $\|LZ_n\|_p \longrightarrow 0$, $\|Z_n\|_{2,p} = 1$. Then $\|LZ_n\|_2 \longrightarrow 0$, and as

$$\int LZ_nZ_n \le -\|\nabla Z_n\|_2^2 + \|(|A|^2+|B|^2)^{1/2}\|_\infty \|\nabla Z_n\|_2\|Z_n\|_2$$

$$+\int CZ_nZ_n \le (r-1+\frac{\kappa}{\lambda_1})\|\nabla Z_n\|_2^2,$$

we conclude that $\|\nabla Z_n\|_2 \longrightarrow 0$. By Poincaré's inequality, we obtain that $\|Z_n\|_2 \longrightarrow 0$ and hence $\|\Delta Z_n\|_2 \longrightarrow 0$. As the lemma holds for $L = \Delta$ and any $1 < p < \infty$ (see [GT]), then $\|Z_n\|_{2,2} \longrightarrow 0$, and hence $\|Z_n\|_{1,p} \longrightarrow 0$. This shows that $\|\Delta Z_n\|_p \longrightarrow 0$, a contradiction. □

REFERENCES

[ABMR] Amster P, Borgna J.P, Mariani M.C, Rial D.F: Solutions to the mean curvature equation for nonparametric surfaces by fixed point methods. Revista de la Unión Matemática Argentina, Volumen 41, 3, 1999. pp 15-21.

[ADNM] Amster P, De Nápoli P, Mariani M. C: Solutions to the prescribed mean curvature equation with Dirichlet conditions by variational methods. Proyecciones, Vol. 18, n: 2, pp. 155-164.

[AM] Amster P, M. C. Mariani M.C: The prescribed mean curvature equation with Dirichlet conditions. To appear in Nonlinear Analysis, Theory, Methods, and Applications.

[BC] Brezis H, Coron J:Multiple solutions of H-systems and Rellich's conjecture, Comm. Pure Appl. Math. 37 (1984), 149-187.

[GT] Gilbarg D, Trudinger N. S : Elliptic partial differential equations of second order, Springer- Verlag (1983).

[H] Hildebrandt S: On the Plateau problem for surfaces of constant mean curvature. Comm. Pure Appl. Math. 23 (1970) 97-114.

[S] Struwe M: Plateau 's problem and the calculus of variations, Lecture Notes Princeton Univ. Press (1988).

[W] Wang Guofang: The Dirichlet problem for the equation of prescribed mean curvature, Analyse Nonlinéaire 9 (1992), 643-655.

[We] Wente H.C: The differential equation $\Delta X = 2HX_u \wedge X_v$ with vanishing boundary values, Proc.Amer.Math.Soc 50, (1975),131-7.

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