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A VARIANT OF COLLATZ'S CONJECTURE OVER BINARY POLYNOMIALS

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Abstract. We study a natural analogue of Collatz's conjecture for polynomials over $\mathbb{F}_2$.

1. Introduction

The Collatz conjecture (also known as the Syracuse problem) is one of the most famous unsolved problems in number theory. It concerns sequences of positive integers defined by the following rule: starting from any positive integer n, if n is even then replace it by $n/2$, and if n is odd then replace it by $3n+1$; see [9]. The conjecture asserts that every such sequence eventually reaches the cycle $4 \to 2 \to 1$.

It is natural to search for functional analogues of this problem over polynomials in $\mathbb{F}_q[x]$, where certain aspects sometimes become easier to analyse. Several authors have proposed and studied such analogues; see, for example, [2, 1, 3, 7, 8] (this list is not exhaustive). One general framework, introduced by Matthews [8], fixes polynomials $K, D \in \mathbb{F}_q[x]$ with $\gcd(K, D) = 1$ and defines the next term T of a current polynomial S by

$$T = \frac{K \cdot S - R}{D},$$

where R is chosen congruent to Kr modulo D for some residue r in a complete residue system modulo D. This guarantees that $T \in \mathbb{F}_q[x]$.

When q is even, many papers concentrate on the particularly simple case $q = 2$, $K = x$ (that could easily imply the case $K = x+1$), and residues $\{0, 1\}$ modulo x. These choices make x play the role of the integer 2. In most of these settings the resulting dynamical system is relatively tame: trajectories either cycle or diverge in a controlled way, and when convergence occurs, the number of steps is bounded by a polynomial in the degree of the starting polynomial.

We propose to study a significantly more difficult variant, still over $\mathbb{F}_2[x]$, in which the role of the integer 2 is played by the product $x(x+1)$. This is motivated by the natural isomorphism $\mathbb{F}_2[x] \simeq \mathbb{F}_2[x+1]$ (substituting $x \mapsto x+1$) and by our earlier work [6], where powers of 2 in $\mathbb{Z}$ (2^{a+b}) were systematically replaced

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by the products $x^a(x+1)^b$ in $\mathbb{F}_2[x]$. That analogy proved fruitful for the study of "Mersenne polynomials" $x^a(x+1)^b+1$ (analogues of 2^k-1) and for classifying perfect polynomials with respect to the sum-of-divisors function $\sigma(A)=\sum_{d|A} d$ (see [4, 5]).

In the present paper we define a Collatz-type iteration on $\mathbb{F}_2[x]$ that simultaneously removes factors of both x and $x+1$. A nonzero polynomial $A \in \mathbb{F}_2[x]$ is called *even* if $\gcd\big(A, x(x+1)\big) \neq 1$, and *odd* otherwise (following [4]). The first odd polynomial different from 1 is $M_1 = x^2+x+1$. Thus the natural analogue of the map $n \mapsto 3n+1$ on odd integers is the map $A \mapsto M_1A+1$ on odd binary polynomials.

Given a nonzero $A_0 = A \in \mathbb{F}_2[x]$, we define valuations

$$\begin{aligned} \mathrm{val}_x(S) &= \text{highest power of } x \text{ dividing } S, \\ \mathrm{val}_{x+1}(S) &= \text{highest power of } x+1 \text{ dividing } S, \end{aligned}$$

and we decompose $S = x^{\mathrm{val}_x(S)}(x+1)^{\mathrm{val}_{x+1}(S)}S_1$ with S_1 odd. The Collatz transformation then generates three sequences:

$$\begin{aligned} A_0 &= A, \\ A_1 &= \text{the odd part of } A_0, \\ A_2 &= M_1A_1+1, \\ A_3 &= \text{the odd part of } A_2, \\ &\vdots \\ A_{2k} &= M_1A_{2k-1}+1, \\ A_{2k+1} &= \text{the odd part of } A_{2k}, \end{aligned}$$

together with the corresponding integer sequences

$$(a_{2k})_k = (\mathrm{val}_x(A_{2k}))_k \quad \text{and} \quad (b_{2k})_k = (\mathrm{val}_{x+1}(A_{2k}))_k.$$

An optimistic binary-polynomial Collatz conjecture would assert:

Conjecture 1.1. *For every nonzero $A \in \mathbb{F}_2[x]$ there exists $m \in \mathbb{N}$ such that for all $k \geq m$,*

$$A_{2k} = x(x+1) \quad \text{and} \quad A_{2k+1} = 1.$$

We prove that every trajectory is finite and eventually reaches precisely this configuration.

Theorem 1.2. *Let $A \in \mathbb{F}_2[x] \setminus \{0\}$. The sequences defined above terminate after finitely many steps. More precisely, there exists $m \in \mathbb{N}$ (depending on A) such that the even-polynomial sequence ends with x^2+x and the odd-polynomial sequence ends with 1:*

$$(A_2, A_4, \ldots, A_{2m}) = (\ldots, x^2+x), \qquad (A_1, A_3, \ldots, A_{2m+1}) = (\ldots, 1).$$

The total number of odd polynomials appearing (the length r_A) satisfies $r_A \leq 2^{\deg(A)-1}$.

The proof of this theorem appears in Section 2.

The exponential bound $2^{\deg(A)-1}$ is clearly suboptimal and far worse than the polynomial-in-degree bounds obtained in the literature for the simpler choices $K = x$ or $K = x + 1$. This deterioration is expected: by simultaneously stripping factors of both x and $x + 1$, our iteration explores a much richer dynamical landscape. Nevertheless, extensive computations up to degree 35 (reported in Section 3) strongly suggest that the true length r_A is still bounded by a low-degree polynomial in $\deg(A)$. The same section analyses special families (e.g. starting polynomials of the form x^n+x+1) and provides further evidence for this phenomenon.

2. Proof of Theorem 1.2

First of all, we need some general results on the numbers of odd and even polynomials of a given degree $d \geq 2$. We let $\mathcal{P}_d$ denote the set of all polynomials of degree d and consider:

$$\begin{aligned} &\mathcal{P}_d^{0,0} := \{S \in \mathcal{P}_d : S(0) = 0\}, \quad \mathcal{P}_d^{0,1} := \{S \in \mathcal{P}_d : S(0) = 1\}, \\ &\mathcal{P}_d^{1,0} := \{S \in \mathcal{P}_d : S(1) = 0\}, \quad \mathcal{P}_d^{1,1} := \{S \in \mathcal{P}_d : S(1) = 1\}, \\ &\mathcal{O}_d := \{S \in \mathcal{P}_d : S \text{ is odd}\} = \mathcal{P}_d^{0,1} \cap \mathcal{P}_d^{1,1}. \end{aligned}$$

We have $\mathcal{P}_0 = \mathcal{O}_0 = \{1\}$ and $\mathcal{O}_1 = \emptyset$.

Lemma 2.1. *The following statements hold:*

(i) *The four sets $\mathcal{P}_d^{0,0}$, $\mathcal{P}_d^{0,1}$, $\mathcal{P}_d^{1,0}$, and $\mathcal{P}_d^{1,1}$ all have the same cardinality 2^{d-1}.*

(ii) *The set $\mathcal{O}_d$ contains exactly 2^{d-2} polynomials if $d \geq 2$.*

Proof. (i) By the bijective map $S \mapsto S+1$, we have $\#\mathcal{P}_d^{0,0} = \#\mathcal{P}_d^{0,1}$ and $\#\mathcal{P}_d^{1,0} = \#\mathcal{P}_d^{1,1}$. Similarly, the bijection $S(x) \mapsto S(x+1)$ gives $\#\mathcal{P}_d^{0,0} = \#\mathcal{P}_d^{1,0}$. It remains to note that $\mathcal{P}_d$ is a disjoint union of $\mathcal{P}_d^{0,0}$ and $\mathcal{P}_d^{0,1}$, and that $\#\mathcal{P}_d = 2^d$.

(ii) By induction on d. The case $d = 2$ is trivial since $\mathcal{O}_2 = \{x^2 + x + 1\}$. Now, suppose that $\#\mathcal{O}_s = 2^{s-2}$ for $2 \leq s \leq d-1$. We observe that

$$\mathcal{P}_d^{0,1} = \{(x+1)^s S_1 : 0 \leq s \leq d,\ s \neq d-1,\ S_1 \in \mathcal{O}_{d-s}\}.$$

Hence,

$$\#\mathcal{P}_d^{0,1} = \#\mathcal{O}_d + \#\mathcal{O}_{d-1} + \cdots + \#\mathcal{O}_2 + \#\mathcal{O}_0.$$

Therefore, $2^{d-1} = \#\mathcal{O}_d + 2^{d-3} + \cdots + 1 + 1 = \#\mathcal{O}_d + 2^{d-2}$, and so $\#\mathcal{O}_d = 2^{d-2}$. □

For $k \geq 0$, we put $d_k := \deg(A_{2k})$ and $\ell_k := \deg(A_{2k+1})$. We then obtain the following lemmas.

Lemma 2.2. *We have $a_0, b_0 \geq 0$ and $a_{2k}, b_{2k} \geq 1$ for any $k \geq 1$.*

Proof. If $k \geq 1$, then both x and $x+1$ divide A_{2k}, because, for $t \in \{0, 1\}$,

$$A_{2k}(t) = 1 + M_1(t) A_{2k-1}(t) = 1 + 1 = 0.$$ □

Lemma 2.3. *We have $\ell_{k+1} \leq \ell_k \leq \deg(A)$, $d_k = \ell_{k-1} + 2$, and $d_k = \ell_k + a_{2k} + b_{2k}$.*

Since $(\ell_k)_k$ is a nonnegative and nonincreasing sequence, we obtain the following corollary.

Corollary 2.4. *Both sequences* $(\ell_k)_k$ *and* $(d_k)_k$ *are convergent. We have*

$$\lim_{k\to\infty} \ell_k = \ell \quad \textit{and} \quad \lim_{k\to\infty} d_k = d, \textit{ where } d = \ell + 2.$$

Corollary 2.5. *There exists* $m \geq 0$ *such that for any* $k \geq m$ *one has*

$$\ell_k = \ell, \quad d_k = d, \quad a_{2k} = b_{2k} = 1.$$

Proof. The convergent sequence $(\ell_k)_k$ takes its values in the finite set

$$\{0, 1, \ldots, \deg(A)\}.$$

Hence it is eventually constant. □

Corollary 2.6. *For any* $k \geq m$ *we have* $\deg(A_{2k}) = d$ *and* $\deg(A_{2k+1}) = \ell$.

Corollary 2.7. *There exists a positive integer* $t \leq \deg(A)$ *such that the polynomials* $A_{2(m+t)}$ *and* $A_{2(m+t)+1}$ *satisfy*

$$A_{2(m+t)} = A_{2m}, \quad A_{2(m+t)+1} = A_{2m+1}.$$

Proof. For any $k \geq m$, the polynomial A_{2k} (resp. A_{2k+1}) lies in the finite set of polynomials of degree d (resp. ℓ). □

Proposition 2.8. *For any* $k \geq m$ *we have* $A_{2k+1} = 1$*, so that* $\ell = 0$ *and* $t = 1$.

Proof. For $k \geq m$ we have $a_{2k} = b_{2k} = 1$. Hence, the Collatz transformations give the following system:

$$\begin{cases} M_1 A_{2m+1} + (1 + M_1) A_{2m+3} = 1, \\ M_1 A_{2m+3} + (1 + M_1) A_{2m+5} = 1, \\ \quad \vdots \\ M_1 A_{2m+2t-3} + (1 + M_1) A_{2m+2t-1} = 1, \\ M_1 A_{2m+2t-1} + (1 + M_1) A_{2m+2t+1} = 1. \end{cases}$$

Since $A_{2m+2t+1} = A_{2m+1}$, we obtain a linear system of t equations with coefficients in $\mathbb{F}_2[x]$ and t unknowns: $A_{2m+1}, \ldots, A_{2m+2t-1}$. Its matrix C is circulant with first row $[M_1, 1 + M_1, 0, \ldots, 0]$. The right-hand side is the transpose of $[1 \ldots 1]$.

By expanding along the first column of C, we find

$$\det(C) = {M_1}^t + (1 + M_1)^t,$$

which is nonzero.

Thus, this system admits the unique solution $(1, \ldots, 1)$. □

Corollary 2.9. *The even and odd sequences* E *and* O *are, respectively,*

$$E = [A_2, \ldots, A_{2m-2}, x^2 + x] \quad \textit{and} \quad O = [A_1, \ldots, A_{2m-1}, 1].$$

Moreover, both E *and* O *contain* $m + 1$ *elements, with* $m + 1 \leq 2^{\deg(A)-1}$.

Proof. We have just seen that $\ell = 0$ and $t = 1$. Thus, $d = 2$, $A_{2m+1} = 1$, and $A_{2m} = x^2 + x$. The odd sequence O contains at most all odd polynomials of degree $\deg(A)$, all odd polynomials of degree $\deg(A) - 1, \ldots$, together with the polynomials $x^2 + x + 1$ and 1.

Thus, by Lemma 2.1 (ii), we obtain

$$m + 1 \leq 2^{\deg(A)-2} + 2^{\deg(A)-3} + \cdots + 2 + 1 + 1 = 2^{\deg(A)-1}. \qquad \square$$

3. Computations

After 7 months and 3 days of computation with PARI/GP, we were able to determine, for each n from 1 to 35, the maximal length $f(n)$ of sequences (A_j) that begin with a polynomial of degree n:

$[n, f(n)]$	$[[1,0],[2,1],[3,2],[4,3],[5,4],[6,8],[7,10],[8,11],[9,12],[10,16]]$
	$[[11,18],[12,20],[13,22],[14,24],[15,28],[16,32],[17,36],[18,38]]$
	$[[19,40],[20,42],[21,46],[22,52],[23,54],[24,55],[25,60],[26,62]]$
	$[[27,66],[28,67],[29,70],[30,74],[31,76],[32,78],[33,84],[34,88]]$
	$[[35,92]]$

From these computations, it appears plausible that $f(n)$ is bounded above by a polynomial in n, thus improving upon our exponential bound in Theorem 1.2.

For each integer n from 2 to 38, we also computed (with PARI/GP, in about 5 months and 8 days) the number $g(n)$ of sequences (A_j) that (a) begin with some odd polynomial $A_1 = A$ of degree n, and (b) have the property that all elements $A_1, A_3, \ldots, A_{2h+1}$ remain of degree n, and that the sequence is maximal with this property:

$[n, g(n)]$	$[[2,1],[3,1],[4,2],[5,2],[6,3],[7,3],[8,4],[9,4],[10,5]]$
	$[[11,5],[12,6],[13,6],[14,7],[15,7],[16,8],[17,8],[18,9]]$
	$[[19,9],[20,10],[21,10],[22,11],[23,11],[24,12],[25,12],[26,13]]$
	$[[27,13],[28,14],[29,14],[30,15],[31,15],[32,16],[33,16],[34,17]]$
	$[[35,17],[36,18],[37,18],[38,19]]$

From these values of n, one may conjecture that $g(n) = \lfloor n/2 \rfloor$. If this conjecture holds, we can bound above the length r_A of the sequence by a polynomial in n as follows:

$$r_A \leq 2(n/2) + 2((n-1)/2) + \cdots + 2(1/2) = \tfrac{n(n+1)}{2}.$$

This improves upon the exponential upper bound of Theorem 1.2. However, for example, when $n = 21$, so that $n(n+1)/2 = 231$, an explicit polynomial A with $g(21) = 10$ has length $r_A = 32$, while $f(21) = 46$.

Concerning some special trinomials, after treating many cases by direct computation (with Maple), we may state some examples, as well as further conjectures, in the next subsections.

3.1. **Example.** In this section, we take $A := T_n = (x^2+x+1)^n + 1 = {M_1}^n + 1$ for $n \geq 1$, so that A is even. Put $n = 2^r u$, where $r \geq 0$ and u is odd. We have

$$A = (M_1 + 1)^{2^r} \cdot (M^{u-1} + \cdots + M + 1)^{2^r}.$$

Hence, the first polynomial in the odd sequence is $A_1 = (M^{u-1} + \cdots + M + 1)^{2^r}$.

On the other hand, if $n \geq 2$, then there exists a unique positive integer r such that $2^{r-1} < n \leq 2^r$. Thus, we may write $n = 2^r - j$, with $0 \leq j \leq 2^{r-1} - 1$.

Hence, we establish the following conjecture:

Conjecture 3.1. *Let $A = {M_1}^{2^r - j} + 1$ where $r \geq 1$ and $0 \leq j \leq 2^{r-1} - 1$. Then, the length of the odd sequence of A is $j + 1$.*

For instance, below we show the odd-degree sequences for $T_n = {M_1}^n + 1$, with $n \in \{9, \ldots, 16\}$, so that $n = 2^4 - j$ with $0 \leq j \leq 7$. Here, all lengths are smaller than $n = \deg(T_n)/2$:

n	Sequence associated with T_n	Length
9	$[16, 16, 16, 16, 16, 16, 16, 0]$	8
10	$[16, 16, 16, 16, 16, 16, 0]$	7
11	$[20, 16, 16, 16, 16, 0]$	6
12	$[16, 16, 16, 16, 0]$	5
13	$[24, 24, 24, 0]$	4
14	$[24, 24, 0]$	3
15	$[28, 0]$	2
16	$[0]$	1

3.2. **Other small lengths.** We now give the odd-degree sequences for $T_n = x^n + x + 1$, $U_n = x^n + x^{n-1} + 1$, and $S_n = x^n + x^7 + x^3 + 1$, with $n \in \{31, 32, 33, 34\}$. Here, all lengths are smaller than $\deg(A)/2 + 2$. Note that S_n is even.

n	Sequence associated with T_n	Length
31	$[31, 29, 24, 24, 16, 16, 16, 16, 0]$	9
32	$[32, 28, 24, 24, 16, 16, 16, 16, 0]$	9
33	$[33, 31, 30, 30, 28, 28, 28, 28, 24, 24, 24, 24, 24, 24, 24, 24, 0]$	17
34	$[34, 31, 30, 30, 28, 28, 28, 28, 24, 24, 24, 24, 24, 24, 24, 24, 0]$	17

n	Sequence associated with U_n	Length
31	$[31, 28, 24, 24, 16, 16, 16, 16, 0]$	9
32	$[32, 31, 30, 30, 28, 28, 28, 28, 24, 24, 24, 24, 24, 24, 24, 24, 0]$	17
33	$[33, 31, 30, 30, 28, 28, 28, 28, 24, 24, 24, 24, 24, 24, 24, 24, 0]$	17
34	$[34, 33, 30, 30, 28, 28, 28, 28, 24, 24, 24, 24, 24, 24, 24, 24, 0]$	17

n	Sequence associated with S_n	Length
31	$[30, 28, 28, 28, 28, 28, 24, 24, 0]$	9
32	$[28, 27, 24, 23, 16, 16, 16, 16, 0]$	9
33	$[32, 31, 22, 22, 16, 16, 16, 16, 0]$	9
34	$[32, 32, 30, 30, 27, 27, 27, 25, 24, 24, 24, 24, 24, 24, 24, 24, 0]$	17

We have two conjectures for $A = x^n + x + 1$ when $n \geq 4$.

Conjecture 3.2. *Let s be the greatest positive integer such that $n - 2^{s+1} \geq 0$. Then, for any positive integer $t \leq s - 1$, the odd sequence contains 2^t polynomials which have the same degree d_t. In particular, $d_1 = \deg(A_5) = \deg(A_7)$ and $d_2 = \deg(A_9) = \deg(A_{11}) = \deg(A_{13}) = \deg(A_{15})$.*

Conjecture 3.3. *Let s be the greatest positive integer such that $n - 2^{s+1} \geq 0$. Then, both the even and odd sequences of A have length $2^s + 1$.*

Conjecture 3.3 follows from Conjecture 3.2. Indeed, from Corollary 2.9, the sequence of the odd polynomials is

$$[A_1, A_3, A_5, A_7, \ldots, A_{2m-1-2^s}, \ldots, A_{2m-3}, A_{2m-1}, 1],$$

of length $m + 1$. One has, by Conjecture 3.2,

$$m + 1 = 1 + 1 + 2 + 2^2 + \cdots + 2^{s-1} + 1 = 1 + (2^s - 1) + 1 = 2^s + 1.$$

3.3. **More small lengths.** Here, the lengths are all equal to $\deg(A) + 1$. Define

$$P_1 = x^{14} + x^{10} + x^9 + x^8 + x^3 + x^2 + 1,$$
$$P_2 = x^{14} + x^{12} + x^9 + x^6 + x^4 + x^3 + 1,$$
$$P_3 = x^{14} + x^{13} + x^9 + x^8 + x^6 + x^5 + x^3 + x^2 + 1.$$

Observe that P_1 and P_2 have the same odd-degree sequences.

Polynomial	Sequence	Length
P_1	$[14, 14, 13, 13, 12, 12, 10, 10, 9, 8, 7, 5, 4, 3, 0]$	15
P_2	$[14, 14, 13, 13, 12, 12, 10, 10, 9, 8, 7, 5, 4, 3, 0]$	15
P_3	$[14, 14, 13, 13, 11, 11, 10, 9, 8, 7, 5, 5, 4, 3, 0]$	15

3.4. **Some technical remarks.** We let $\overline{S}$ denote the polynomial obtained from $S \in \mathbb{F}_2[x]$ by replacing x with $x + 1$. We also consider the reciprocal S^* of S, defined as

$$S^*(x) = x^{\deg(S)} \cdot S\left(\tfrac{1}{x}\right).$$

It is easy to see that the Collatz sequences of $\overline{A}$ are obtained from those of A by applying the operation $S \mapsto \overline{S}$. However, for A^* this is not generally true. For example, if $A = x^8 + x^3 + 1$, then the odd sequence is $[8, 7, 5, 5, 4, 3, 0]$, whereas for

$A^* = x^8 + x^5 + 1$, one obtains $[8, 6, 6, 0]$. Nonetheless, if A is of the form $x^n + x + 1$, then the odd-degree sequence associated with A^* is different (in general), but still regular enough (see Section 3.2, with $U_{32} = {T_{32}}^*$).

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